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N.V. PHILIPS' GLOEILAMPENFABRIEKEN

EDITED BY THE RESEARCH LABORATORY OF N.V. PHILIPS' GLOEILAMPENFABRIEKEN, EINDHOVEN, HOLLAND

THE PRODUCTION OF SHARP FLUORESCENT SPOTS IN CATHODE RAY TUBES

Summary. The problem of producing sharply-defined light spots on a fluorescent screen with the aid of a beam of electrons is studied in this article, use being made as far as practicable of the analogy between light-rays and the cathode rays.

Introduction

During recent years, the cathode ray (or Braun) tube which was originally only employed for physical experiments has been adapted to permit of its use for various technical purposes. The earliest use of the cathode ray tube was in the cathode-ray oscillograph in which the motion of a sharp spot of light on a fluorescent screen was studied. The television apparatus, which has already been described in the first number of this Review [1]), is one of the latest applications of the cathode ray tube. In this apparatus the image is produced on the fluorescent screen by causing a spot of light (or scanning beam), whose intensity is subject to rapid fluctuations, to travel at great speed over the whole picture-surface. A third important application of the cathode ray tube is the electron microscope, in which the cathode ray throws an enlarged image of the hot cathode surface on the fluorescent screen. *Fig. 1* reproduces a series of photographs of the crystal structure of a cathode surface, which were obtained by Dr. W. G. Burgers with the aid of an electron microscope.

In the various applications of the cathode ray tube, the specific requirements were such as to necessitate a closer investigation and improvements of various details. It appears desirable, therefore, in this introductory article to review the principles governing the construction and method of operation of the cathode ray tube.

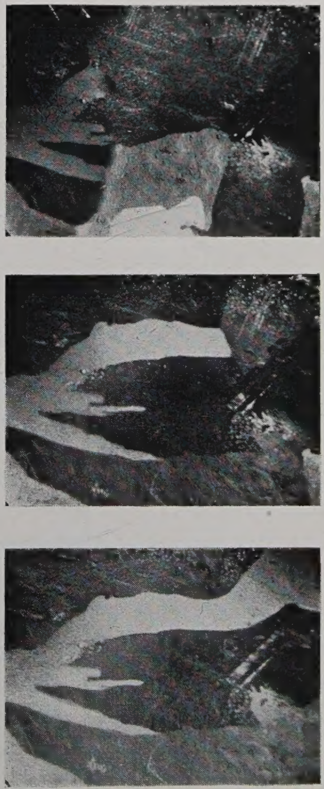

Fig. 1. Series of pictures of a cathode surface produced by cathode rays (magnification 15 times).

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[1]) Philips techn. Rev. 1, 16, 1936.

Fig. 2 shows the construction of a gasfilled cathode ray tube of very simple design. The hot cathode K is surrounded by the cylinder C, which is given a negative bias of such magnitude that owing to attraction by the gas ions the electrons emitted from the cathode are focussed as far as possible to a single point (*fig.* 3). If a potential difference is applied to the pair of plates P_1 and P_2, the beam of electrons is deflected. Owing to the small rout-time of the electrons from the cathode to the screen, the beam of electrons can respond to electrical oscillations much more quickly than mechanical oscillographs. Only at frequencies higher than 10^8 per sec (i.e. at a wavelength of less than 3m) in high vacuum tubes the cathode ray does no longer follow the impressed oscillations.

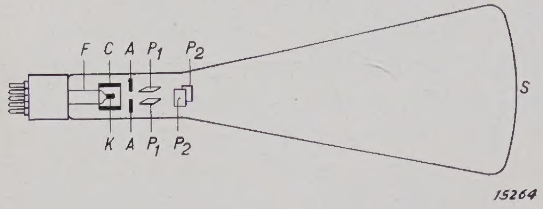

Fig. 2. Diagrammatic sketch of a gas-filled cathode ray tube. K = cathode. F = leading-in wires. C = Wehnelt cylinder. A = anode. P_1 and P_2 = deflecting plates. S = screen.

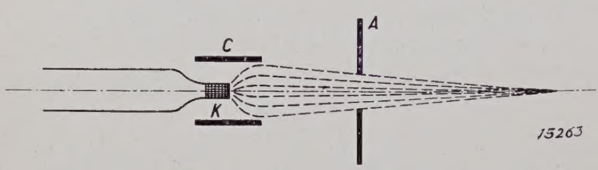

Fig. 3. Image of the cathode obtained with the aid of the Wehnelt cylinder C. Owing to the electric field of C and the attraction of the gas ions the electrons emitted by the cathode are concentrated on a point of the axis forming an image of the cathode.

Electron Lenses

Although the cylinder C (fig. 2) is quite effective for focussing the beam of electrons in gasfilled tubes, sharply defined cathode images can indeed only be obtained with the aid of more complex electrical arrangements. This is the only successful method in high-vacuum tubes. To visualise clearly the method of operation of such additional electric fields, we will discuss the electrical system by analogy with optical systems. We therefore also speak of electron lenses, etc.

The path of light-rays in geometrical optics is governed by Fermat's principle, according to which the path of these rays between two points is such that the time for passing from one point to the other is a minimum. In a medium with a refractive index n, the velocity of light is c/n, where c is the speed of light in vacuo. Fermats' principle may be expressed mathematically as follows:

$$\int dt = \int \frac{n}{c}\, ds = \frac{1}{c} \int n\, ds = \textit{minimum} \qquad (1)$$

In order to describe the path of a beam of electrons, we must start from the fundamental laws of point mechanics. These laws may be summarised in the principle of minimum action, according to which the space-integral of the product of the mass m and the velocity v (mv = impulse p) between any two points of the space traversed is smaller for the actual path taken than for any other conceivable path terminating at the same two points with the same velocities. It is presupposed here that the velocity of the body at every point in its path is as great as required by the energy theorem. We thus obtain the following condition for a minimum:

$$\int p\, ds = m \int v\, ds = \textit{minimum}\ . \quad . \qquad (2)$$

Comparing equations (1) and (2), it is seen that the velocity v of the electrons in electronic optics plays exactly the same part as the refractive index n in geometrical optics. If, with the aid of metal lattices, lenticular areas in cathode ray tubes (*fig. 4*) are raised by a constant amount of potential above the surrounding space, the electrons will travel within such areas at greater velocities than outside them. The velocity within the electron lens is determined by the potential difference the electrons passed through:

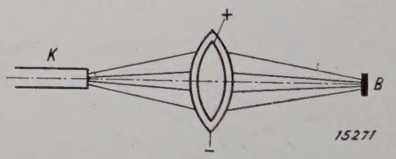

Fig. 4. Electronic lens of wire lattice. K = cathode. B = image. The wire lattices are electrical double layers. At the small distance between them the electrons are much accelerated or retarded; outside and inside the electron lens their velocity is a constant.

$$e\, V + \frac{1}{2}\, m\, v^2 = \textit{const.}$$

V being the potential and e, m [2]) and v respectively the charge, the mass and the velocity of the electron.

The lenticular area thus acts on the electrons in an analogous way to an ordinary optical lens.

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[2]) $e = 4{,}77.10^{-10}$ e. s. u.
$m = 9.10^{-28}$ gr.

The practical production of lenses by means of wire lattices has, however, certain disadvantages as the electrons are scattered at the lattice wires.

Immersion Systems for Electrons

Of an efficient cathode ray tube it is not only required that the image of the cathode on the fluorescent screen shall be as small as possible, but also that the optical system and the screen shall be sufficiently far apart (fig. 2). This large intervening distance is necessary so that the fairly small angular deflections, which the pair of plates P_1 and P_2 impart to the beam of electrons, are rendered visible on the screen on a magnified scale. On the fluorescent screen we thus obtain an image O' of small area as well as a beam of small angle Ω'. On the other hand, to obtain a sufficient intensity it is necessary to make the area of the cathode O as well as the angle of emission Ω of the electron beam as large as possible.

According to a theorem in geometrical optics, the product of the area O, the angle Ω of the beam and the square of the refractive index n^2 is the same for both image and object, thus:

$$O' \, \Omega' \, n'^2 = O \, \Omega \, n^2 \quad . \quad . \quad (3)$$

A corresponding formula also applies in electronic optics, except that the refractive index n is replaced by the velocity v of the electrons. The requirements to be met by the cathode ray tube set forth in the previous section can obviously only be satisfied if the electrons striking the screen have a much greater velocity than those leaving the cathode. In optics, systems in which the object and image are embedded in media with different refractive indices are termed immersion systems. The same nomenclature may be applied in electronic optics to systems in which the velocities of the electrons at the cathode and at the screen are not the same.

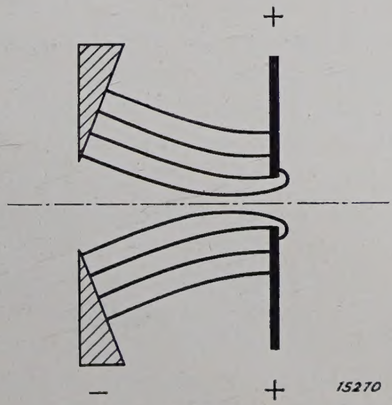

Fig. 5. Electrostatic field of an immersion system for electrons. It can be deduced from the shape of the lines of force that the electrons moving from left to right are accelerated; so they pass through the aperture of the anode at a high velocity.

For light rays the refractive indices vary only between 1 and 2; with cathode rays, however, any desired ratio between the velocities corresponding to the refractive indices, can be obtained by accelerating the electrons sufficiently. In this way both the angle of the beam and the image of the cathode can be made as small as desired. The beam of electrons is focussed by means of electric fields with a rotational symmetry instead of by wire lattices. An example of this is shown in *fig. 5*. The focussing action of such electric fields can be readily deduced from the configuration of the lines of force. *Fig. 6* shows the arrangement of the electronic lenses in the Philips 3951/3952 cathode ray tubes. The Wehnelt cylinder is provided with a metal plate which has a small aperture just in front of the hot cathode. Owing to the negative bias of the Wehnelt cylinder, the beam of electrons on admission into the first anode cylinder is already subject to preliminary focussing. The electric field between the two anode cylinders ensures accurate concentration of the electron beam.

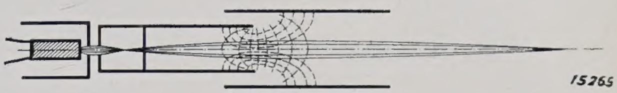

Fig. 6. Arrangement of an electronic lens in the Philips 3951/3952 cathode ray tube. Lines of force: — — — — —; equipotential surfaces: — . — . — . —; path of electrons: ——

Magnetic Electron Lenses

Besides electrical systems, magnetic systems such as a homogeneous magnetic field with the same direction as the electron beam, can also be employed for focussing the cathode ray. For the action of such a magnetic field there is no optical analogy. If a slightly divergent beam of electrons travels in the longitudinal direction of a magnetic field, the electrons are deflected by Lorentz forces which are perpendicular to the direction of motion of the electrons and to the magnetic field. Under the action of these Lorentz forces, the electrons describe elongated spiral paths whose axes are in the direction of the magnetic field. As these spiral paths (*fig.* 7) all have the same pitch, the electron beam is concentrated to a single focus.

We shall analyse this process in greater detail. Take an electron with a velocity v which makes a small angle a with the direction of the magnetic field, so that the radial component of the velocity is $v \sin a$, while the axial component is almost equal to v. The projection of the path of the electron on a surface perpendicular to the axis of the magnetic field is then a circle of radius r. The centripetal force

$$\frac{m\,v^2\,\sin^2\alpha}{r}$$

required to produce this motion is provided by the magnetic field H, which is therefore equal to $H\,(e/c)\,v\,\sin\alpha$ (c is the velocity of light). This gives the radius of the circle as:

$$r = \frac{c\,m\,v}{eH}\,\sin\alpha \quad . \; . \; . \; . \; (4)$$

The radii of the circles are therefore proportional to the velocities of the electrons v and inversely proportional to the magnetic field strength H.

At a given field-strength, all circular paths will be described in the same time, viz:

$$\tau = \frac{2\pi r}{v\,\sin\alpha} = 2\pi\,\frac{m\,c}{eH} \quad . \; . \; (5)$$

During this time of motion the electrons will traverse the same distance s in an axial direction:

$$s = v\tau = 2\pi\,\frac{mcv}{eH} \quad . \; . \; . \; (6)$$

The spiral paths thus all have the same pitch (fig. 7).

With magnetic focussing very good results are obtained. It is also used in the electron microscope with which the photographs of fig. 1 were made.

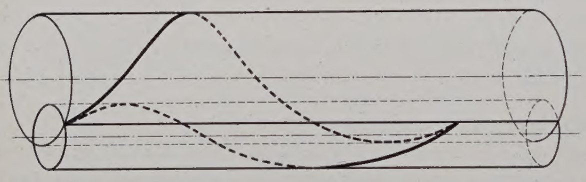

Fig. 7. Spiral paths described by electrons in a homogeneous magnetic field. The cylindrical surfaces enclosing these spiral paths are shown solid for the sake of elucidation. The axis of the system is the common generator of all cylinders.

Distortion of the image

As a rule light-rays of different colours are refracted to different degrees at the surfaces bounding two media, so that two rays of light of different colours coming from the same point are usually not focussed in exact coincidence, and so-called chromatic aberration is obtained. The corresponding distortion in electronic optics is obtained with non-homogeneous beams, a beam being defined as non-homogeneous when the constituent electrons have different velocities and are therefore deflected to different degrees in the electric field. The electrons being emitted by the cathode with various velocities, the electron beam is always non-homogeneous. The difference between the velocities of the electrons can, however, be kept within sufficiently narrow limits by accelerating the electrons already close to the cathode to such a degree that their velocity is large compared with different emission velocities.

If in geometrical optics the apertures through which the ray passes are not made too small, then the rays emitted from one point in the axis will be combined at different points according as they travel close to the axis or make a large angle with it. Distortion of the image due to this factor is termed spherical aberration. The corresponding analogy in electronic optics is the aberration of homogeneous beams of electrons with a finite angle of emission, but in practice such distortion is small as the apertures employed are not very large.

If a plane perpendicular to the axis appears as a curved surface in the image, we have what is called curvilinear distortion, which is of little trouble in electronic optics as we are dealing with beams of great depth of definition (cf. the section of space-charge phenomena, fig. 10) in which the surface of the image does not occupy a strictly defined position.

A beam of light-rays which does not travel parallel to the axis of the lens will as a rule not be concentrated at a focus, but along two focal lines perpendicular to each other and perpendicular to the axis of the lens at different distances from the lens. This form of distortion is termed the astigmatism of inclined beams. In optical systems not accurately centred, astigmatic distortion of beams parallel to the axis also occurs. In electronic optics this phenomenon may be very troublesome, for the beam of electrons here gives rise to two lines perpendicular to each other, which are thrown on the screen at different potentials. To obtain sharply-defined images astigmatic distortion must therefore be kept as small as possible. It has been found that the deviations from rotational symmetry and the error in centring the cathode and the Wehnelt cylinder must not exceed 0.01 mm. In other parts of the optical system the dimensions in question are greater, so that the geometrical and centring errors may also be somewhat greater (about 0.1 mm). Furthermore, non-homogeneities of the surface of the cathode may also cause deviations in the rotational symmetry of the beam of electrons, resulting in the production of so-called cathodic astigmatism. Fortunately this distortion usually disappears in course of time as the surface of the cathode becomes sufficiently homogeneous after a few hours' heating.

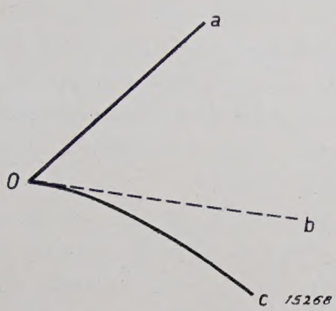

Fig. 8. Rotational distortion projected on a surface perpendicular to the axis. O is the axis, *a* is a line cutting the axis perpendicularly; *b* is its rotated image according to equation 6; *c* is the figure produced by rotational distortion.

When using magnetic lenses, for which there is no analogy in geometrical optics, distortion of the image is also obtained, but for whose elucidation no optical analogy can be of any assistance. In the case of rays which only deviate slightly from the direction of the axis, we obtain in a homogeneous magnetic field H a specific rotation of eH/cmv per unit of length in the direction of the axis. If the electrons are also accelerated during their passage through the magnetic coil, their total rotation in the magnetic field is the integral of this specific rotation along the path of the electron. Non-homogeneous magnetic fields also enable well-defined images to be obtained, whereby the image is not only rotated but also magnified or reduced. If the rays make a larger angle with the axis, their rotation may differ from that of the rays travelling almost parallel to the axis, and for this reason straight lines perpendicular to the axis are frequently shown in the image as curved. In this case we speak of rotational distortion (*fig. 8*).

Deflection Systems

The deflecting fields naturally possess no rotational symmetry but usually an enantiomorphic symmetry, and thus behave as cylindrical lenses in geometrical optics. Care must be taken that the astigmatism caused by them is as small as possible, for if for instance one of the deflecting plates is connected to the anode and an alternating voltage applied only to the other, the astigmatism at the same potential difference between the plates will be much greater than when the two plates are connected up symmetrically with respect to the anode.

If the electrons are deflected to a marked extent, they will approach the edges of the deflecting plates where the field is very non-homogeneous. Deflection will then no longer be proportional to the deflecting potential difference. To avoid this source of error, the ends of the deflecting plates are splayed outwards (*fig. 9*) so that the electrons on leaving the deflecting system can travel along equipotential surfaces for as great a distance as possible.

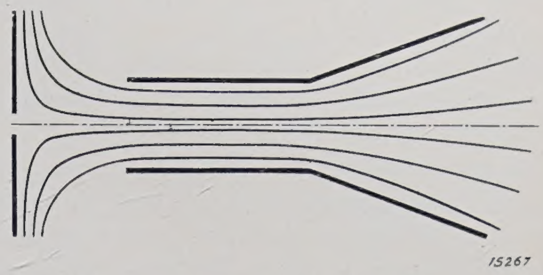

Fig. 9. A deflecting system in which the electrons on emission are almost parallel to the equipotential lines.

Space-Charge Phenomena

The mutual repulsion of the electrons has no corresponding analogy in geometrical optics and the distortion of the cathode ray resulting from this action must therefore be discussed independently. The fluorescent spot owing to the space-charge becomes greater than it should do according to geometrical optics. If in a 2000-volt oscillograph the maximum diameter of the spot is stipulated as 1 mm the greatest permissible current intensity is then 0.1 milliamp. In modern cathode ray tubes for television purposes this current value is reached although only at higher voltages. Perhaps the current strength of the cathode ray could be further increased by using systems with a greater aperture and in which the current density remained sufficiently low up to the neighbourhood of the screen.

With a cathode ray the position of the image obtained is not by any means fixed as precisely as with light-rays, for the beam of electrons in the neighbourhood of its maximum constriction has practically a constant cross-section over a fairly considerable distance (*fig. 10*). A cathode ray has therefore a much greater depth of definition, so that curvilinear distortion (see section on distortion of the image) is of no moment.

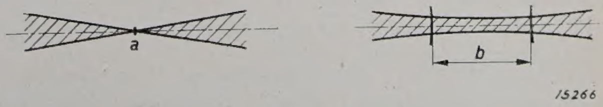

Fig. 10. Production of an image: (a) by light-rays, (b) by a beam of cathode rays. With light-rays the outlines of the beam are straight lines, with cathode rays they are curved by mutual repulsion of the electrons. Therefore these electron beams possess approximately their minimum diameter along a large distance.

The thickness of the graphed line in an oscillogram obtained with a cathode ray tube is also determined by the graphing speed. This phenomenon is termed the "charging spread" and is due to irregularities in the charging of the screen caused by the greater or lower velocity of the scanning cathode ray. Charging produces an electrostatic field in front of the screen which scatters the electron beam.

The electric charge of the cathode ray, in addition to the troublesome effects mentioned above, has fortunately also an advantageous action. The so-called gas concentration which can be obtained in gas-filled tubes, results from the ionisation of the gaseous atoms by the high-speed electrons. In this way the passage of the cathode ray produces a track of positive ions, which, as they are far less mobile than the electrons generated at the same time, remain in their original positions, while the electrons diffuse into the surrounding space. In the path traversed a space-charge field is thus produced which not only counteracts the mutual repulsion of the electrons but is also even capable of focussing the beam of electrons. In gas-filled tubes sufficiently small spots can be obtained without special electric fields.

For the gas concentration gas pressures of 10^{-2} to 10^{-3} mm are required. Neon, argon or mercury vapour are among the gases used for filling the tubes. Cathode ray tubes of this type give satisfactory spots already with a few hundred volts. A disadvantage of the gas filling is the danger of causing deterioration of the cathode by bombardment with positive ions. To prevent this action the anode is so constructed that it isolates the cathode space as far as possible from the deflection space where the majority of the ions are produced.

The focussing action of the space-charge field is closely determined by the radiation intensity. On varying the intensity the equilibrium between the electrons, ions and gaseous atoms in the neighbourhood of the ray is naturally also altered. For television purposes, in which the intensity of the ray varies considerably, it is not a simple matter to obtain satisfactory results with gas concentration. For this reason also, high-vacuum tubes are used for television which are run on several kilovolts, so that owing to the much greater velocity of the electrons their mutual repulsion is not given time to spread out the beam.

Compiled by G. P. ITTMANN.

RELAXATION OSCILLATIONS

Summary. The characteristics of relaxation oscillations are a fixed amplitude (and not an amplitude determined by the initial conditions, as in the case of harmonic vibrations) and a period determined by a relaxation time (i.e. the time-delay of the formation of an equilibrium). This class of oscillations is widely distributed and is met with in the most varied branches of science and technology. They fulfil a special function in the maintenance of harmonic vibrations, a motion with which the engineer is much more closely acquainted. A very useful property of relaxation oscillations from the practical standpoint is the ease with which they can be synchronised. Between the two limiting cases of perfect relaxation oscillations and perfect harmonic motion there are transitional forms which can be produced experimentally as well as derived from a differential equation.

Introduction

In the case of many natural phenomena, progress in our knowledge of them and their subsequent technical application do not date from the time of their discovery but from the time they were given a distinguishing name. The acquisition of a specific name is an indication that the phenomenon in question, although it may have been known already for centuries, has become a focus of interest and that the research worker commencing to study and investigate it in greater detail, now requires some means of readily identifying it.

Relaxation oscillations were given this name [1]) in 1926 when a number of investigators, among which are also those in the Philips Laboratory, began to show a deeper interest in this type of phenomenon. It was soon realized that very many periodic processes in the most divergent branches of science could be interpreted as relaxation oscillations. The newly-christened type of oscillation was also found to be a useful tool to the engineer, which he had unknowingly already been employing for a long time. In the first issue of this Review special applications of these oscillations have already been described in two different articles (J. van der Mark, An Experimental Television Transmitter and Receiver, p. 16; D. M. Duinker, Relay Valves as Timing Devices in Seam Welding, p. 11).

The mechanism and most important properties of those systems which are subject to relaxation oscillations will be discussed here.

A Special Relaxation Oscillation as Example

To demonstrate without preamble the particular class of processes which enter into consideration here, we shall start with the discussion of an example which has indeed been known for a long time, viz. the intermittent discharge of a glow discharge tube having a condenser connected in parallel.

The circuit of this system is shown in *fig. 1*. B is a discharge tube of any type, which has a specific ignition voltage V_1 and a specific extinction voltage V_2.

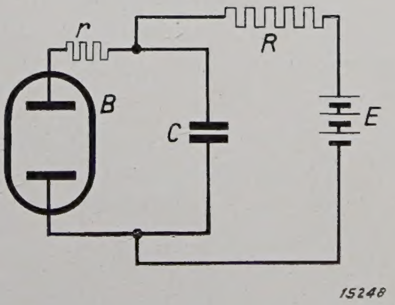

Fig. 1. A system which can perform relaxation oscillations: The condenser C is charged from battery E through the suitably-rated resistance R. The discharge tube B has initially an infinitely high resistance. However, as soon as the potential of the condenser exceeds the ignition voltage of B, the tube becomes ignited, i.e. its resistance drops abruptly to a low value r. The condenser discharges itself rapidly through this low resistance until its potential has dropped to the extinction voltage of B, whereupon the resistance of the tube again becomes infinite and the same cycle recommences.

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[1]) By B. van der Pol in the Tijdschr. v. h. Nederl. Radio Genootsch. **3,** 25, 1926, (cf. also Phil. Mag. **2,** 978, 1926). About the same time, E. Friedländer, Arch. f. Elektrotechnik **17,** 1, 1926, termed these oscillations "Kippschwingungen", which name has been adopted in Germany.

The tube commences to conduct current as soon as the potential across its terminals exceeds V_1. The condenser C, which is charged from a battery E through the high resistance R, is discharged through the low resistance r of the tube B as soon as it has reached the potential V_1. The potential of the condenser on discharge through the low resistance r drops rapidly to the extinction voltage V_2, at which the tube allows no further current to pass and the discharge again ceases. The condenser is then recharged and its potential again rises from V_2 to V_1 when the same cycle is repeated. The potential V of the condenser thus fluctuates as shown in *fig. 2*.

It is seen that the oscillation obtained is very different from an ordinary sinusoidal one. In the curve in fig. 2, as in other forms of relaxation oscillations (see fig. 7c), characteristic features are the steep discontinuities which occur at the end of each period corresponding to the sudden discharge. The length of the period is therefore mainly determined by the time required for charging the condenser (from the potential V_2 to V_1); this interval of time is determined by the resistance R and the capacity C. An interval of time of this kind, which quite generally represents the delay with which an equilibrium is arrived at (in the present case the equilibrium between the potential of the condenser and the battery voltage) is termed a relaxation time. In the class of oscillations which are discussed here, the vibration period is always determined by a relaxation time, hence the term "relaxation oscillations" proposed by van der Pol.

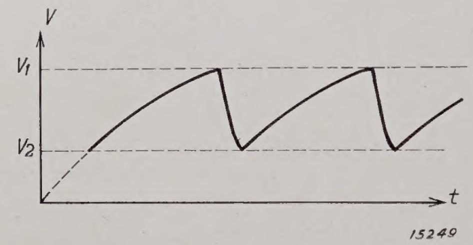

Fig. 2. Variation of the potential V of the condenser in the circuit in fig. 1. V_1 is the ignition voltage, V_2 the extinction voltage. The curve represents a relaxation oscillation.

Apart from this rather superficial difference between relaxation oscillations and the well-known harmonic (sinusoidal) motion, there are other reasons which justify relaxation oscillations being regarded and investigated as independent phenomena. It might be advanced against this that a relaxation oscilation similar to every other periodic phenomenon can be formally compounded from various harmonic motions by means of a Fourier series. But in contradistinction to a harmonic motion, the most characteristic property of a relaxation oscillation is, indeed, not the periodicity of the process. In our example we saw how a typical aperiodic process, viz. the charging of a condenser, can be continually repeated, linked up by interposed intervals of discharging of the condenser. The rhythm in which these sequential operations take place is determined by the rate of supply from the energy source E through the resistance R; by disconnecting the source E we can interrupt the charging process at any desirable moment and then again continue it from the same point at any subsequent time. This system of oscillation therefore exhibits no preference for any characteristic frequency, which latter can be readily influenced by extraneous factors. On the other hand, the amplitude of the relaxation oscillation is fixed; the reversal from charging to discharge of the condenser and inversely always takes place at the ignition voltage or at the extinction voltage of the discharge tube, the system continually fluctuates between these two voltages.

Application of Relaxation Oscillations for Maintaining Harmonic Oscillations

Let us now consider the conversion of energy which takes place with a relaxation oscillation and with a harmonic oscillation respectively. In the example of a relaxation oscillation discussed above, we had an energy accumulator (the condenser) to which energy was supplied from a constant source (the battery). The system also contained a unit (the discharge tube) which at a certain high "tension" of the energy accumulator suddenly commences to dissipate energy (converting it to heat) until the potential of the accumulator has dropped to a specific low value: the dissipation of energy then ceases and the same cycle commences afresh. The net result is, therefore, that the system absorbs and dissipates a definite quantity of energy from a constant energy source, and this process can be continued indefinitely.

In a system of harmonic vibrations, we always have two energy accumulators, e.g. a mass m in which kinetic energy can be accumulated and a spiral spring c in which potential energy can be stored (*fig. 3a*), or a condenser C for electrical energy and a self-induction L for magnetic energy (*fig. 3b*). During the motion energy is transformed periodically from the one form to the other. A dissipation of energy, which occurs in all practical cases (friction, electrical resistance), results in the gradual decay of the oscillation. Hence to obtain a sinusoidal vibration lasting a long time, we must make up the energy lost. But here the striking fact must be considered that from a source of constant energy we cannot directly apply energy to a system of harmonic oscillations in order to maintain the vibratory motion. This may be readily seen with the aid of the differential equation for harmonic motion, e.g. in the mechanical system (fig. 3a):

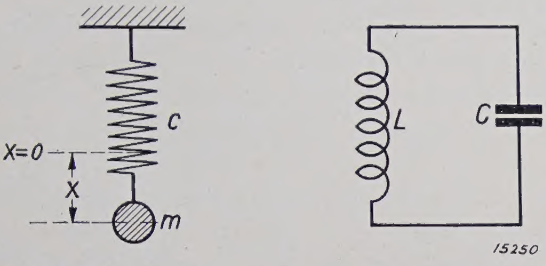

Fig. 3. Examples of systems carrying out harmonic vibrations.
a) Mechanical system consisting af a mass m attached to a spring with a retracting force c.
b) Electrical system comprising a condenser C and a self-induction L connected in series.

$$m\,\frac{d^2x}{dt^2}+a\,\frac{dx}{dt}+cx=0 \quad . \quad . \quad . \quad (1)$$

where x is the deviation of the system from its position of rest (i.e. the amplitude) and the three terms give in the order shown the inertia (accumulator m or L), the friction (dissipation of energy) and the retracting force (accumulator c or C), which at every instant of time are in dynamic equilibrium. If for the moment we neglect damping, the solution of (1) is a sine function (*fig. 4*) whose frequency is fixed, but whose phase and amplitude can have any values, being only governed by the initial conditions (the magnitudes of x and dx/dt at time $t=0$). If a constant external force K is applied to the system, equation (1) becomes:

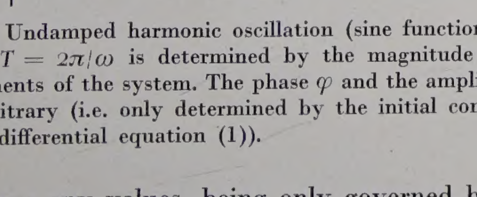

Fig. 4. Undamped harmonic oscillation (sine function). The period $T=2\pi/\omega$ is determined by the magnitude of the components of the system. The phase φ and the amplitude A are arbitrary (i.e. only determined by the initial conditions of the differential equation (1)).

$$m\,\frac{d^2x}{dt^2}+a\,\frac{dx}{dt}+cx=K \quad . \quad . \quad . \quad (1a)$$

The energy imparted to the system by this means is determined by the integral of the force over the path traversed:

$$\textit{Work done} = \int K\,dx = K\int dx$$

But if we substitute for x the sine function expressing the oscillation to be maintained, the integral is equal to zero over each complete period of the sine [2]). In order that the value of the integral shall not become zero, i.e. energy is actually supplied to the system, we must apply a force which is not constant but is an arbitrary function of the time, provided only that this function contains a component synchronising with x (e.g. to be determined by Fourier analysis), i.e. a sinusoidal function similar to x. In the majority of cases the sources of energy available furnish constant forces: for instance, gravity, wind pressure or a current of water, the E.M.F. of a battery, etc. In order to introduce the requisite sinusoidal component here, we must intercept the constant time function of the force at regular intervals, so that the influx of energy is alternately free and blocked. But this is just what relaxation oscillations are able to do. We can therefore place between the constant energy source and the damped system of harmonic vibrations a system of relaxation oscillations in order to supply the requisite energy in fractions as required.

This principle is indeed actually employed in practice. The simplest example of maintained harmonic oscillations which suggests itself is the mechanism of a clock. Being constrained to a definite frequency, harmonic oscillations are ideal for time measurement; thus, a pendulum (in a wall clock) or a balance wheel (in a watch) or a piezo-electric quartz lamina (in some modern astronomical clocks) is allowed to oscillate and the time measured by counting the number of oscillations. To maintain these oscillations, so that as a result of damping they do not rapidly decay and cease altogether, we provide a source of constant energy, e.g. the wound spring in a watch. Through the lever A

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[2]) For the constant force merely causes a displacement of the position of rest about which the system oscillates; this follows from the differential equation (1a) which on making the substitution $x'=x-K/c$ can be written as:

$$m\,\frac{d^2x'}{dt^2}+a\,\frac{dx'}{dt}+cx'=0.$$

This is again the same equation as (1), except that the position of rest is now $x'=0$, i.e. $x=K/c$, instead of $x=0$.

(anchor) of the escapement (*fig. 5*) the balance wheel O is given a fresh impulse at the end of each half-period of its oscillation by the driving spring uncoiling a very little way, i. e. a small part of its potential energy is transmitted as twist to the gear mechanism and the anchor. The lever A of the escapement performs a typical movement of the relaxation oscillation type; it moves to and fro between two fixed terminal positions to the right and left (stops s), the reversal in direction in this case being produced itself through the coupling with the harmonic oscillation [3]).

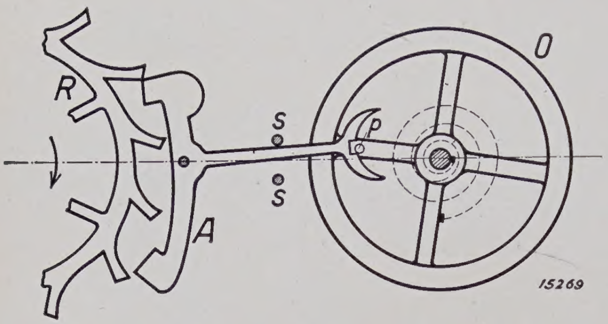

Fig. 5. Escapement of a watch. The harmonic vibrations of the balance wheel O are maintained by a periodic supply of energy through the anchor A which performs a relaxation oscillation. In detail this process is as follows: As soon as the balance wheel during oscillation approaches its middle position (maximum velocity), the pin p coming from above strikes against the fork of the anchor A, which up to this moment rested against the upper pin s. The anchor is turned slightly, and its upper claw slides along the tooth of the escapement wheel R against which it rested; when the tooth has been released (this instant is shown in the figure), the scape wheel turns through the space of one tooth, under the tension of the driving spring, in the direction of the arrow (being again immediately engaged by the lower claw of the anchor) and now accelerates the motion of the anchor by means of the lifting "surface" at the top of the tooth in such a way that the anchor tilts into its opposite terminal position (the lower stop s), its fork at the same time giving a push to the pin p of the balance wheel in the direction of its oscillation. Hence, in this moment energy is imparted to the harmonic system by the spring. The anchor remains in its lower terminal position until the balance, after passing through half a swing, again initiates the same operations in the opposite direction.

A further example is a tuning fork whose harmonic oscillations are maintained by a similar mechanism as used in the electric bell: this is also a relaxation oscillation (the energy accumulator charged and discharged being here the magnet coil) [4]).

The sound in an organ pipe is also produced in a similar way: the column of air in the pipe performs a harmonic oscillation, which is created and maintained by a current of air blown over a tongue. Air eddies are produced behind the tongue which grow to a certain size, then become unstable and release the tongue. This is repeated periodically and is another example of a relaxation oscillation which maintains a harmonic oscillation.

The Synchronising of Relaxation Oscillations

In the above we have seen the important practical difference in behaviour between systems of harmonic and relaxation oscillations with respect to a constant force. We shall now see how these two systems would react to a periodic, e.g. sinusoidal, external force.

Under the action of an external periodic force, a harmonic system becomes subject to what are called forced vibrations, with a frequency equal to that of the external force, but with an amplitude which is very small if there is a great difference between the forced and natural frequencies; if, however, the forced vibration frequency approaches the value of the natural frequency, the amplitude of the system becomes large, i.e. "resonance" is obtained.

A system of relaxation oscillations does not exhibit a similar resonance phenomenon. The amplitude is here fixed by the difference between the "potentials" of the energy accumulator at which a reversal of the controlling unit (in our first example the discharge tube) takes place. The external force, however, determines the moment reversal takes place: without appreciably altering the amplitude of the relaxation oscillations it influences its frequency within certain limits.

This is very aptly shown in frequency demultiplication [5]). In the system in fig. 1 a periodic voltage $e \sin \omega t$ (*fig. 6*) is applied in series with the discharge tube. We then find that the time between two ignitions, i.e. a period of relaxation oscillation, favours an integral multiple n ($n = 1, 2, 3, \ldots$) of the periods of the applied voltage: the frequency of the relaxation oscillation becomes synchronised with the n-th part ($1/n = 1/1, 1/2, 1/3, \ldots$) of the applied frequency, viz, that fraction whose value is nearest to the frequency of the "free" (unconstrained) relaxation oscillation. If in fig. 6 the "free" frequency is gradually altered by varying the capacity of the condenser, the frequency of the "forced" vibrations initially remains unchanged, i.e. in step with the fraction of the extraneous frequency with which it has been synchronised; as the deviation becomes greater the frequency jumps to the next fraction to which the "free" relaxation frequency has now approached more closely. In this way van der Pol and van der Mark, (loc. cit.) have succeeded in synchronising relaxation oscillations with 1/200 of the frequency of an applied oscillation.

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[3]) That the escapement through the scapement wheel R at the same time sets in operation a counting mechanism which enables us to read the time on the dial, is of no importance for our considerations. By the way, the "relaxation system" (anchor) could not in this case perform independent oscillations without being coupled to the harmonic system.

[4]) For musical purposes, Niaudet has constructed a clock in which a tuning fork with a frequency of 64 cycles acting as regulator is driven by an escapement (the scapement wheel here makes a complete revolution per second). If this clock loses a second per day, the frequency of the tuning fork is only 63.99926 cycles, as may be readily calculated. With this instrument extremely accurate measurement of pitch is possible by the method of beats.

[5]) B. van der Pol and J. van der Mark, Nature **120**, 363, 1927.

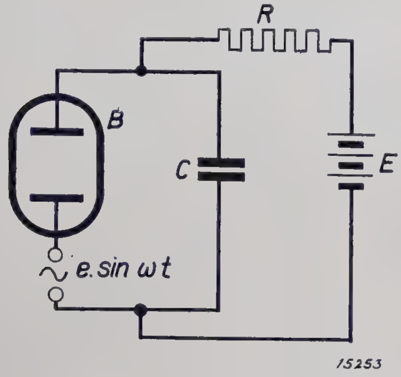

Fig. 6. The same system of relaxation oscillations as shown in fig. 1 (intermittent glow discharge), but here with the addition of a periodic voltage $e \sin \omega t$ which affects the times of ignition and extinction.

Further Examples and Technical Applications of Relaxation Oscillations

We have seen above that in the case of harmonic oscillations a special mechanism is required for the periodic addition of fresh energy. Relaxation oscillations do not require a similar provision. Hence nature and technology provide us with many examples of systems of relaxation oscillations which continue indefinitely in vibration without any special demand being made on the energy source for the maintenance of the motion. In addition no harmonic oscillation is required to control reversal as, for instance, in the case of the clock. We have already met with a typical example in the swelling and self-generating air eddies in a continuous current of air when discussing the organ pipe; the same process is also responsible for the musical sounds of the Aeolian harp and the humming of telegraph wires in the wind. The period of the note produced (pitch) is determined by the duration of growth of an eddy to the unstable size, i.e. by the velocity of the wind and the diameter of the wire, but does not depend on the length and tension of the wires (as with the harmonic vibration of strings, in for example the violin and piano). Many other well-known sounds are also due to relaxation vibrations; the grating of a knife on a plate, the fluttering of a flag, the hammering or singing of a water pipe, etc.

In the biological world, there are probably also many phenomena which can be interpreted as relaxation oscillations, e.g. the beat of the heart. In this periodic process the fact that the heart "beats" already enables us to deduce that it does not perform a sinusoidal vibration but a relaxation oscillation. Actually in the heart three systems of relaxation oscillations are coupled in a specific manner. Van der Pol and van der Mark[6]) by coupling in a similar way three electrical systems of relaxation vibrations have succeeded in reproducing in a most striking manner electrocardiograms as obtained with healthy and diseased hearts.

The opening and closing of blossoms is also frequently a relaxation oscillation which the plants continue to carry out with a definite periodicity when they are removed from all external influences (such as light, heat, etc.) and which can then be synchronised with new light stimuli.

In electro-technology, relaxation oscillations have perhaps been investigated most exhaustively of all branches of science. Apart from the example of the intermittent glow discharge and the electric bell, there is the Wehnelt interrupter, further the combination of a series-wound dynamo which feeds a motor with independent field-excitation, the dynamo commutating periodically; furthermore there is the multi-vibrator of Abraham and Bloch, as well as other circuits with electron tubes with specific characteristics.

Particularly the last-named electrical relaxation oscillations are widely employed in technology (cf. for instance the above-mentioned descriptions already published in the first issue of this Review). Other applications deserving of mention are the use of bi-metal switches for luminous advertising signs and other purposes (the relaxation period is here the time required for heating), the motion of windscreen wipers on motor cars which are driven pneumatically by a type of escapement, and finally the automatic flushing of water closets by cisterns which periodically are filled slowly and then rapidly discharged by means of a syphon.

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[6]) B. van der Pol and J. van der Mark, Phil. Mag. 6 763, 1928.

Mathematical Representation of Relaxation Oscillations

In a previous section we have used the differential equation (1) of harmonic motion. The question arises whether we can also represent relaxation oscillations mathematically, i.e. deduce their properties from the solutions of certain differential equations.

Consider again our first example of the intermittent glow discharge in fig. 1. During the charging of the condenser C from the battery with potential E through the resistance R, the discharge tube has an infinitely high resistance; the condition of equilibrium between the voltages gives us the following differential equation for the condenser current i_1 at this point:

$$Ri_1 + \frac{1}{C}\int i_1 \, dt = E \quad . \quad . \quad . \quad (2a)$$

The solution of this equation is

$$i_1 = \frac{E}{R} e^{-\frac{t}{RC}}$$

For the condenser potential $V = E - i_1 R$ we thus get:

$$V = E\left(1 - e^{-\frac{t}{RC}}\right)$$

This represents the first portion of the curve shown in fig. 2: V increases with a velocity which is determined by the "relaxation time" RC in the exponential function. But as soon as V becomes greater than the ignition voltage V_1 of the tube, its resistance jumps from infinity to the value r. From this instant onwards the discharge current of the condenser i_2 in the opposite direction is defined by the following differential equation (neglecting the small term r/R with respect to unity):

$$r\frac{di_2}{dt} + \frac{1}{C} i_2 = 0 \quad . \quad . \quad . \quad . \quad (2b)$$

which gives for the potential of the condenser

$$V = V_1 \, e^{-\frac{t}{rC}}$$

We have now arrived at the next portion of the curve in fig. 2, the one with a negative slope, until the extinction potential V_2 is reached; the velocity with which this point is reached is again determined by a relaxation period, which is here rC. The total duration of the process, which is continually repeated, is the sum of the two components of which each one is proportional to a relaxation period [7]).

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[7]) It is readily found that the charging period is

$$T_1 = RC \cdot \ln \frac{E - V_2}{E - V_1}$$

and the discharging period

$$T_2 = r\,C \cdot \ln \frac{V_1}{V_2}$$

The whole period is $T = T_1 + T_2$.

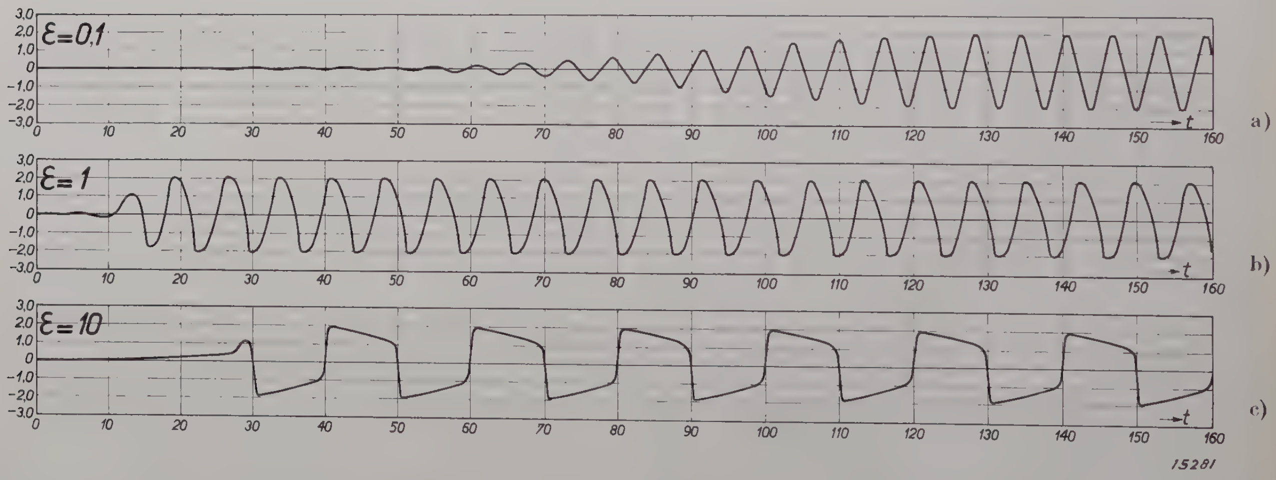


Fig. 7. Graphically found solutions of the differential equation (3) for three different values of the parameter ε.

a) For $\varepsilon << 1$ the solution is a gradually-developing sinusoidal vibration with a terminal amplitude of 2.

b) For $\varepsilon = 1$ we get a distinctly distorted vibration, which more rapidly reaches the terminal amplitude.

c) For $\varepsilon >> 1$ the solution brings out the typical properties of a relaxation oscillation. A to and fro movement between two terminal positions is shown, neither such positions nor the zero position being stable themselves. On closer analysis it is found that the period is proportional to a magnitude physically equivalent to a relaxation period.

Corresponding to the two entirely different processes of which it is made up, the relaxation oscillation is here represented by two differential equations (2a and 2b) which have alternate application.

B. van der Pol has succeeded in deducing mathematically from a single differential equation the behaviour of the multi-vibrator and other systems, which perform relaxation oscillations and were referred to above under technical applications. This equation after certain simplifying transformations can always be written in the form:

$$\frac{d^2x}{dt^2} - \varepsilon\,(1 - x^2)\,\frac{dx}{dt} + x = 0 \quad \ldots (3)$$

Compared to differential equation (1) for harmonic vibrations, equation (3) differs in respect to the non-linear "damping" term containing the coefficient ε. The solution of (3) (which is obtained by a graphical method) proves to be closely dependent on the magnitude of ε (cf. *figs. 7a, b and c*). If $\varepsilon << 1$, e.g. $\varepsilon = 0.1$ (fig. 7a) we obtain an almost sinusoidal vibration with a gradually increasing amplitude. At $\varepsilon = 1$ (fig. 7b) the sine function has become distinctly distorted and the final amplitude is attained much quicker. At $\varepsilon >> 1$, e.g. $\varepsilon = 10$ (fig. 7c), a vibration is obtained whose essential properties agree with those of what we have defined as a "relaxation oscillation" in fig. 2. We have an amplitude determined by two fixed terminal positions (independent of the energy consumption), as well as the typical discontinuities (the attainment of instability); also the period (numerically approximately equal to ε) can be shown to be determined by a magnitude which is essentially a relaxation period. The symmetrical form of the function with respect to the abscissa (fig. 7c) is only due to the simple form of the damping term; if we add to it an asymmetrical term so that the damping term becomes

$$\varepsilon\,(1 - \beta x - x^2)\,\frac{dx}{dt}$$

the solution also becomes asymmetric, resembling still more fig. 2.

This far-reaching concordance between the behaviour of the general type of relaxation oscillation discussed above and the properties of the solution of equation (3) for $\varepsilon >> 1$ makes it not unjustifiable to regard equation (3) as the mathematical expression of the phenomenon of the "relaxation oscillation". From it the essential characteristics of systems of relaxation oscillations can be deduced and predicted, although care must be taken that conclusions of further import are not drawn. For those cases for which the differential equation (3) has been deduced physically (multivibrator, etc.) it has been observed that a continuous transition to sinusoidal vibrations is possible; and these transition forms have actually been realised experimentally in the cases in question. From the equation (3), however, no direct conclusions can be drawn regarding the behaviour of, for instance, the intermittent glow discharge with respect to these transition forms.

Compiled by S. GRADSTEIN.

OPTICAL MODEL EXPERIMENTS FOR STUDYING THE ACOUSTICS OF THEATRES

By R. VERMEULEN and J. DE BOER.

Summary. In the auditoriums of theatres where one of the primary considerations is the perfect audibility and intelligibility of the spoken word, a short period of reverberation is essential; this, however, should not cause too great a reduction in the "useful sound intensity". The distribution of loudness is examined in this paper with the aid of optical model experiments, in which the source of sound is replaced by a small lamp and the walls of the theatre simulated in a three-dimensional model by walls with a suitable coefficient of reflection. This method was applied on the occasion of the rebuilding of the assembly hall in Philips "Ontspanningsgebouw" (Philips Theatre).

To gain an idea of the acoustics of a hall or theatre during a concert or a theatrical production requires no special knowledge or auxiliary means. On the contrary, the opinions of the uninitiated audience witnessing the show here act as a yardstick. If the audience is not satisfied, it is the business of the expert to establish the cause of such dissatisfaction. But he must also be in a position to determine the potential acoustic characteristics of a hall even before it is built, merely from a study of the plans. In both cases he must make use of measurements and experience with existing halls and from these deduce with the aid of calculations and designs the information required. In addition, the behaviour of the sound waves in the structure in question can also be determined by means of a small scale model of the hall. A new method will be described here for carrying out such experiments with models, which has been applied in the Philips Laboratory, inter alia, to assist in the reconstruction of the assembly hall in Philips "Ontspanningsgebouw" (Theatre) at Eindhoven.

Acoustic Properties of a Hall

The acoustic characteristics of a hall or theatre are almost completely determined by the reflection of the sound waves at the walls. The sound which becomes audible to the audience can be resolved into various components, viz:

1) The direct sound, i.e. the sound waves reaching the audience without reflection at one of the walls or other surface;
2) The "useful" sound, which in addition to the direct sound also includes every sound wave striking the ear within $^1/_{15}$ of a second after the arrival of the direct sound. Useful sound has been so named because it has been found that all these contributed sounds aid in making the spoken word audible and intelligible, and in practice cannot be distinguished from the direct sound.
3) Reverberation, which includes all sound persisting after the emission from the sound source has ceased. Reverberation thus embraces the "useful" sound and all contributed sound waves which reach the ear after $^1/_{15}$ of a second.
4) For the sake of completion echoes will also be included here. An echo comprises a reflected sound wave which, owing to the peculiar shape of a part of the reflecting walls, predominates above the reverberation and can be distinguished separately because of its high intensity.

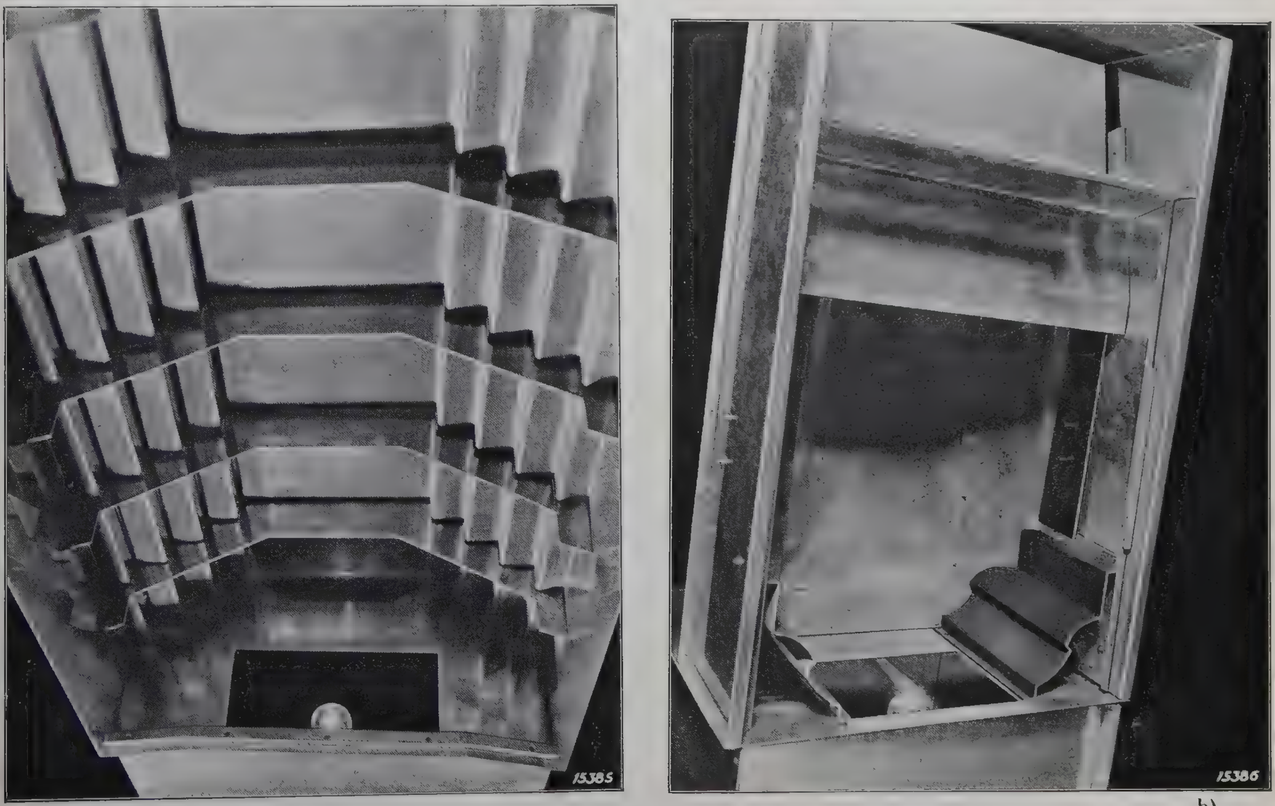
The reverberation period is defined as the time taken by the intensity of sound in a sound-filled space to decay to a millionth part of its initial intensity after emission from the sound source has ceased. In the case of musical productions it has been found that there is an optimum value of the reverberation period at which the music is heard to the best effect. For the spoken word, specific reverberation periods also give optimum results, signifying that at these values maximum intelligibility is obtained. That an optimum is actually obtained here is due to the fact that the useful sound constitutes a part of the reverberation, so that with too short a reverberation period the useful sound intensity becomes too weak; if, on the other hand, the reverberation period becomes too long the successive syllables overlap and the intelligibility again deteriorates.

In the ideal theatre the whole sound radiated from the source would reach the audience as useful sound. Except for the case where the speaker stands in a suitable spot and is surrounded by a sound reflector, this acoustic ideal cannot be even approximately realised. A certain quantity of sound will always be "wasted", reaching the audience only after many reflections, i.e. after a long delay, and in the form of reverberation will only have an adverse effect on intelligibility. Even if for a particular theatre the useful sound intensity has been found to be sufficiently large, it is still necessary to keep the reverberation period within reasonable limits by the provision of sound-absorbing materials.

Optical Model Experiments

The hall of Philips Theatre was primarily intended for theatrical performances for which it was essential to obtain a sufficient intensity of useful sound. On reconstruction of the auditorium it appeared desirable to obtain an insight into the magnitude and distribution of the useful sound intensity in the old theatre and in the new one for which plans had been drawn up. For this purpose model experiments were carried out in which the source of sound was replaced by a light-source and the resulting light intensity determined directly for different parts of the auditorium.

The model of the hall is made of sheet aluminium (see *figs. 1a and b*), and a small lamp is used as a "sound-source" which can be placed in different positions. About 50 per cent of the light-rays impinging on the aluminium walls is reflected so that after three reflections the intensity of a light-ray is already reduced to about 10 per cent of its initial value and thus will no longer contribute much to the total intensity. The luminous intensity at each point in the model is, therefore, determined by the interaction of only a few reflected rays,



a) b)

Fig. 1. Models for examining the "useful" intensity of sound in a hall. The source of sound is represented by a small lamp, shown at the bottom of the photograph. The walls of the model are made of aluminium which reflects about 50 per cent of the light-rays falling on it, so that, in accordance with the definition of the "useful" intensity, a ray after only a few reflections can just contribute to the illumination of the opal glass representing the audience. (In the pictures this glass has been removed in order to show the interior; in the upper half of fig. 1b, however, the opal glass representing the seating on the balcony is visible. Sound-absorbing surfaces in the hall are blackened in the model, e.g. the right-hand wall in fig. 1b. The brightness of the opal glass (see figs. 4, 8 and 11) represents the distribution of the useful intensity of sound.

a) Model of hall of Philips "Ontspanningsgebouw" (Theatre) before reconstruction.

b) Model of the same hall after reconstruction on the plans of Prof. Witzmann, Vienna.

i.e. the resultant luminous intensity is actually a measure of the useful sound intensity. In the model the area occupied by the audience was represented by an opal glass plate with a low reflecting power 1). The brightness of this plate which may be observed and photographed from the outside, gives a picture of the distribution of sound intensity over its area.

How far is our assumption that an analogy exists between sound and light waves justified? We all know from experience that sound waves per se exhibit an entirely different behaviour to light waves. Light is propagated rectilinearly, undergoes simple reflection and casts deep shadows. From a superficial glance, none of these properties appears to be shared by sound waves. Sound travels round corners, does not throw deep shadows and on the whole does not appear to obey the laws of simple reflection to any extent (except in the case of echoes) but always seems to undergo scattered reflection. This difference in behaviour is due to the difference in wave-length; visible light is made up of (ether) waves with a wave-length from 0.4 to 0.76 μ, while audible sound consists of atmospheric waves with a wave-length from 6 cm to 6 m, i.e. 100,000 to 10 million times longer. Waves which strike an obstruction will behave similarly if the dimensions of the obstruction are the same relative to the wave-length. Thus sound waves are reflected at surfaces in exactly the same way as light waves, if the dimensions of the surface are large as compared with the wave-length, although surface inequalities which just permit a true reflection of sound waves may be far greater for these waves than for light waves. It is, therefore, necessary to use a perfectly smooth surface for light, while sound can still exhibit true reflection at fairly rough-finished walls. We can therefore assume that sound in an auditorium is reflected in the same way as the light-rays in our models. It is true this no longer applies to low-frequency sounds, as a hall usually contains many surfaces which are not very large in comparison to these (long) wave-lengths. But the intelligibility of the spoken word depends in a large measure on just the higher socalled characteristic frequencies (thus whispered words in which the low tones in particular are absent can be heard almost equally as well as loudly-spoken words). These frequencies are in the neighbourhood of 1,000 cycles, corresponding to a wave-length of 30 cm. For these important speech frequencies the light model can, therefore, be definitely employed.

Application of the Method

Firstly a model was built of the hall before reconstruction (*figs. 1a, 2 and 3*). The length of the hall was 34 m, its width 16 m and the greatest height 9 m. At an earlier period the rear wall had been completely covered with Akusti-celotex to suppress echoes. The floor sloped a little upwards. The roof was supported by framework trusses and was of stepped construction with the vertical sections of glass, which were closed by roller shutters during performances. The trusses were completely covered by a panelling and thus formed transverse walls 1.8 m high.

On making a closer examination of the distribution of brightness on the opal glass of the model (*fig. 4*), a feature which strikes one immediately is the abundance of light close to the stage, representing the direct sound. Perhaps more striking still is the fact that this intensity diminishes so rapidly to the rear, much more rapidly in fact than should follow from the quadratic diminution in intensity with distance for a point source of light. The

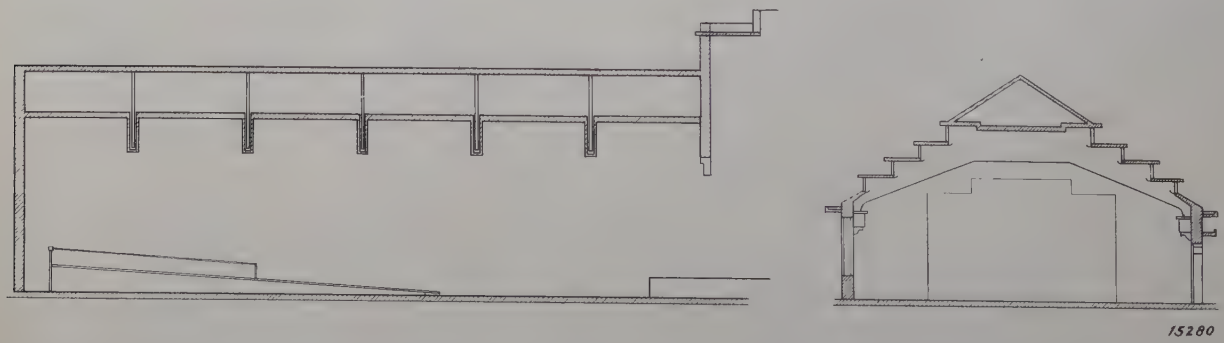


Fig. 2. Longitudinal and transverse sections through the hall of the Theatre before reconstruction. The hall was very low compared to its length. Note the transverse partitions in the roof which are formed by the roof trusses.

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1) The coefficient of absorption of the audience is taken as 0.90 to 0.95.

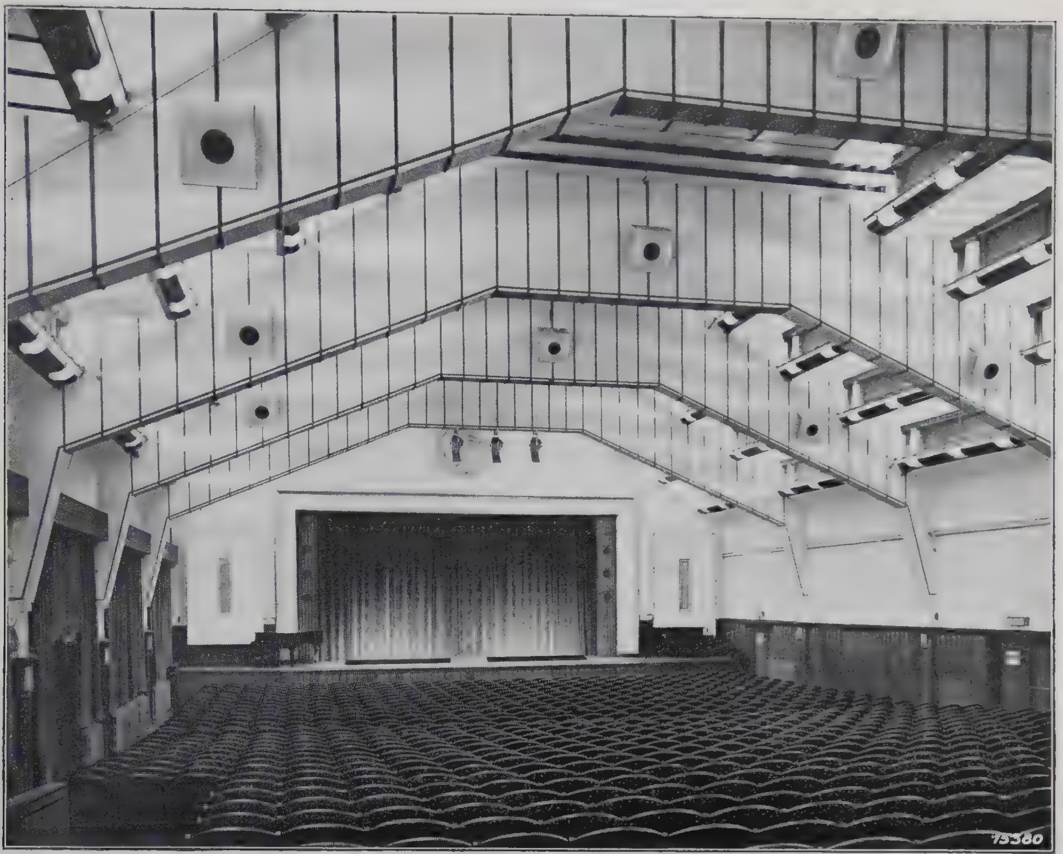

Fig. 3. Auditorium of the "Ontspanningsgebouw" (Theatre) before reconstruction.

reflection at the ceiling, however, which is shown as a trapezoidal area of light in the foreground, has a very high intensity, much greater than that of the direct sound, although the distance from the source of sound along the reflected ray is much greater and in reflection a certain amount of absorption has taken place.

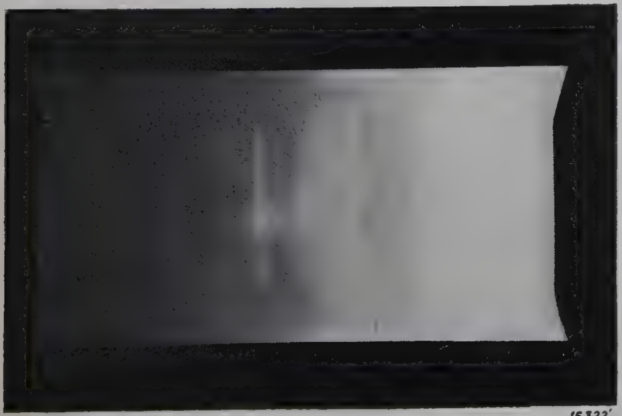

Fig. 4. Photograph of the opal glass in the model of the old hall (fig. 1a). The stage is on the right-hand side. The rapid diminution in illumination intensity towards the left may be noted: at the rear of the hall the useful sound intensity was far too small. The bright trapezoidal spot near the stage is due to reflection at the forepart of the ceiling (see text) which was not screened by the transverse roof trusses.

And yet this phenomenon is easy to account for. The brightness of the opal glass is determined by the quantity of light falling on unit area. The source radiates light with an equal intensity in all directions, so that the intensity of illumination is determined by the solid angle which the unit of surface subtends at the source. As the height of the source above the audience is not very great and the floor slopes only slightly, this solid angle diminishes very rapidly towards the back (*fig.* 5).

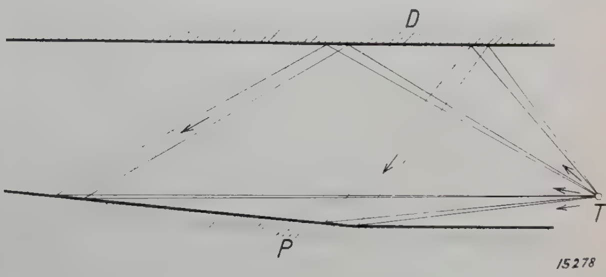

Fig. 5. Effect of reflection at the ceiling. In the normal case of a slightly sloping floor (auditorium P) the solid angle subtended by the audience at the sound source T decreases rapidly towards the rear of the hall. On the other hand, the solid angle subtended by the audience at the mirror image of the sound source above the ceiling D diminishes much more slowly: Reflection at the ceiling contributes more to the sound intensity than the direct sound waves.

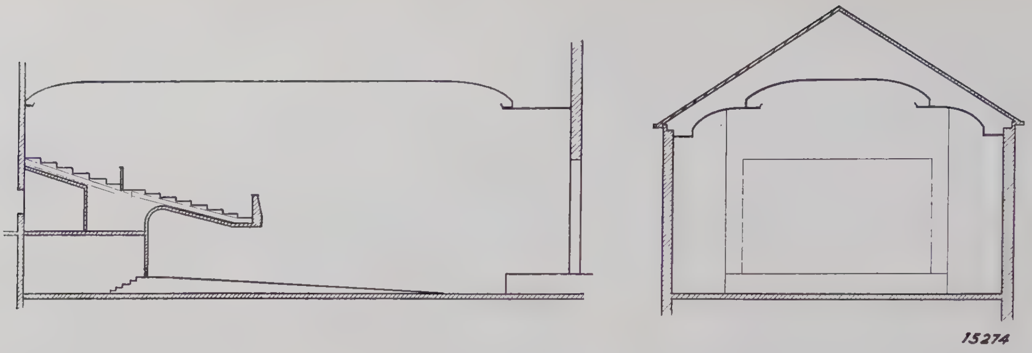

Fig. 6. Longitudinal and transverse sections of the hall in the Theatre after reconstruction on the plans of Prof. Witzmann. The auditorium has been made shorter and higher (cf. fig. 2), and a balcony has been added, the seating accomodation being practically the same as in the old hall.

On the other hand, the solid angle which the unit of surface (in the auditorium) subtends at the mirror image of the source above the ceiling is much greater in spite of the greater distance these are apart. It is thus clear that the audience receives far more energy from the reflected ray than from the direct ray [2]).

The roof trusses prevent the sound reaching the back parts of the hall by reflection at the ceiling, so that in that area only the weak direct sound is received. The rear wall of the hall reflects the sound,

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[2]) It may be argued against this that the sound receiver in the auditorium (the head of a person in the audience) is vertical and the solid angle it subtends at the stage must not be compared with a surface of the same area located on the sloping floor. It could thus be deduced that the sound intensity received by reflection at the ceiling could in no case be greater than the direct sound. This explanation would, however, not be correct: the seating and the audience, particularly in the projection perpendicular to the direction of the direct sound, in their parts have measurements of the order of magnitude of the sound wave-lengths and thus act as a single absorbing surface; or, expressed in other words, the seats or the heads of the audience screen off the sound by their diffraction effect, although one can see optically across the elements of this diffraction "grating". Cf. Békésy, Z. f. techn. Phys. **14**, 6, 1933, and Brillouin, Rev. acoust. **4**, 113, 1935.

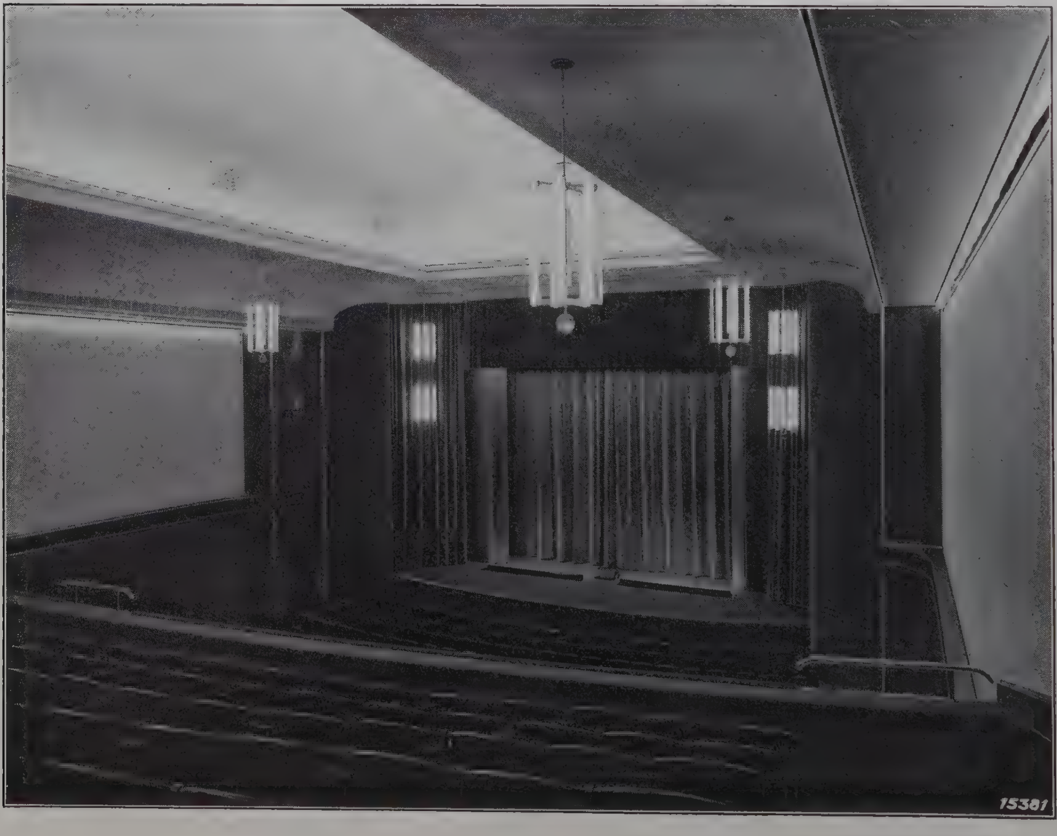

Fig. 7. Auditorium of the Theatre after reconstruction, as seen from the balcony.

not towards the audience, but also towards the ceiling, or rather to the roof trusses, so that this sound is also lost and only becomes audible after a large number of reflections as reverberation. It was, therefore, correct in this case to make the rear wall sound-absorbing.

This model shows in a most convincing manner that in general the largest part of the audible sound comes from the roof, provided the latter is of suitable design. Further confirmation of this is the fact that in open-air theatres the spoken word is much less audible. To remedy this drawback, the benches are arranged on a steep slope or amphitheatre, and it is clear that as a result thereof the available solid angle through which sound is received is considerably increased.

The second model made was of the same hall reconstructed on the plans of Prof. Witzmann of Vienna (*figs. 1b, 6 and 7*). The whole roof has been raised and a ceiling fixed under the trusses. The hall has also been made shorter and a balcony added, which runs further back than the lower part of the hall. The new dimensions are: Width 16 m, height 11 m, length 22 m, and length with balcony 28 m. The seating accomodation has been left practically unchanged. The improved distribution of sound was immediately revealed by the new model (*fig. 8*). The forepart of the auditorium again receives a large percentage of direct sound, and from the middle of the hall reflection at the ceiling comes into play and transmits a satisfactory intensity of sound to the balcony also. The space under the balcony naturally receives no part of this reflected sound and is therefore shown in the model as much darker. How this drawback has been remedied is outlined below.

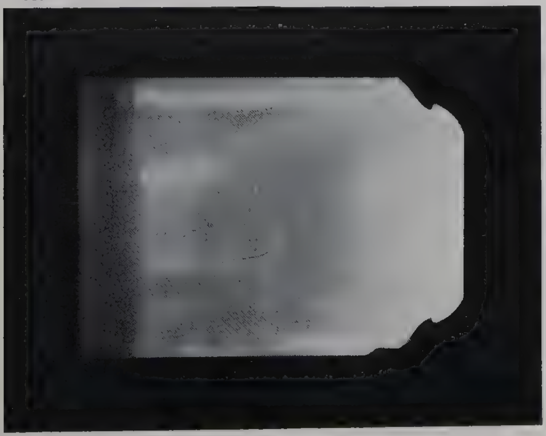

Fig. 8. Photograph of the opal glass in the model of the new hall (fig. 1b). The stage is again on the right-hand side. The distribution of illumination (useful sound intensity) is much more uniform than in fig. 4. The rear part of the hall under the balcony is still fairly dark (see text and fig. 11).

The side walls are panelled and hence absorb sound to some extent. The model and experiments in a "ripple tank" [3]) (*fig. 9*) had already demonstrated that the sound falling on the walls of the auditorium did not have much opportunity to reach the audience as useful sound. After repeated reflection between the walls the bulk of this sound contributes to reverberation. The side-walls thus appeared to be the most suitable places for fixing the panelling, which was necessary to reduce the reverberation period to the desired value (for this purpose a certain degree of absorption at the walls is necessary).

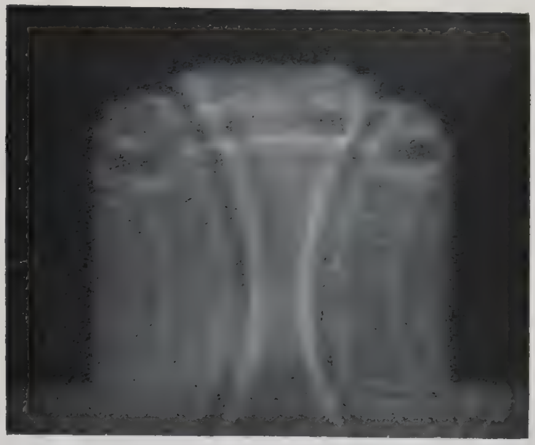

Fig. 9. Registration of waves in a ripple tank model of the cross-section of the new hall. By this means the effect of reflection at the side walls was investigated. It is seen that the waves are in the main reflected to and fro between the walls, so that they do not contribute much to the useful sound intensity but only increase reverberation.

In this way the requirement has been met as far as practicable that the sound shall reach the audience either directly or after one or two reflections (with the largest possible solid angle) and then be absorbed. All other sound has to be absorbed to such an extent that the hall does not possess too small a reverberation period. The hall can therefore also be used for concerts for which a greater blending of sound is required, i.e. the sound-waves also after several reflections must reach the audience with sufficient intensity.

It must still be outlined how the audibility under the balcony was improved. After the optical models had shown that the intensity there was too small, the area in question was examined in greater detail

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[3]) In these experiments a reduced section of the hall is filled with water and the propagation of small artificial water waves is studied.

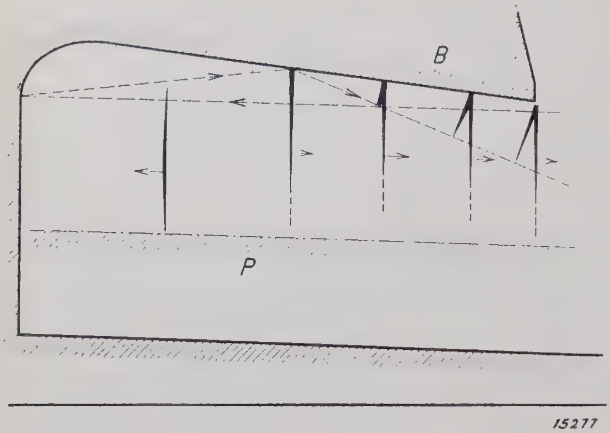

Fig. 10a. Path of a sound wave entering the space under the balcony B and reflected at the vertical rear wall. The wave is so to speak folded back by reflection at the underside of the balcony and again leaves this space without reaching the audience P.

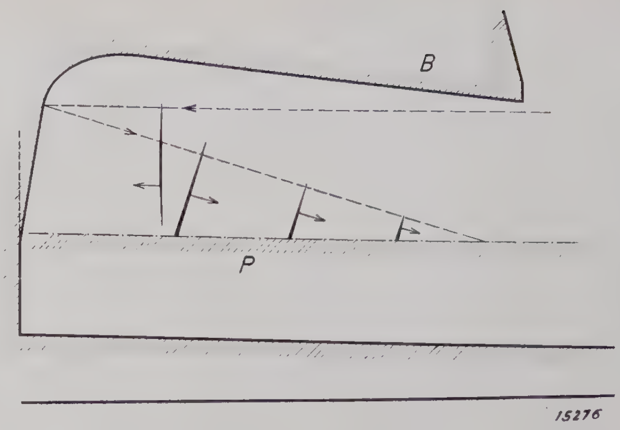

Fig. 10b. The same wave as shown in fig. 10a, but with a wall sloping slightly forward in place of the vertical wall. The entering wave is now immediately reflected to the audience.

in the ripple tank. It was then found that the rear wall reflected an intense wave to the underside of the balcony, but which impinged against the latter at such a small angle of incidence that it was not reflected to the seats under the balcony (*fig. 10a*). The wave was as it were folded up and left the space as a concentrated packet of waves, passing just under the edge of the balcony. This difficulty was very easily remedied: If the rear wall is given a slight slope forward the sound waves are reflected directly to the audience in this part of the hall (*fig. 10b*). Naturally in doing this care must be taken that only the back rows are reached by the wave so that not too long an interval intervenes between the arrival of the direct sound and that of the sound reflected at the wall (echo). In the model the effect of the sloping rear wall was clearly demonstrated (*fig. 11*).

Already during the inaugural performances in the new hall on October 11, 12, and 13, 1935, audibility was found to be very good in all parts of the auditorium; later performances confirmed that at all points of the hall, also under the balcony, the words spoken on the stage were now clearly intelligible and no effort in listening was required whatsoever.

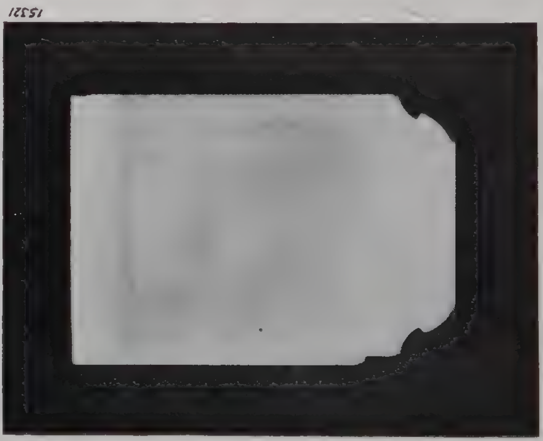

Fig. 11. Photograph of the opa glass in the model, as in fig. 8, but now with the rear wall under the balcony sloping slightly forward. The space under the balcony now also receives a satisfactory intensity of sound.

A HIGH-FREQUENCY FURNACE WITH VALVE GENERATOR

Summary. An induction furnace employed in the foundry of Philips works especially for melting charges of scrap is described. Alternating current of high frequency is required as a power supply for the furnace and is generated with the aid of a transmitting valve which has a useful output of 250 kW.

To be of service to other finishing branches of its works, Philips have for some years been operating their own melting shop and foundry, which for example produces a chrome-iron alloy that can be fused with glass to give a vacuum-tight joint and in view of this property is employed in a large number of apparatus. One of the commonest functions of the melting shop is to melt comparatively small quantities of material under conditions guaranteeing a prescribed analysis within narrow limits of composition. In this work great difficulty is usually experienced with alloying constituents which burn in air, such as the carbon in ferro-alloys, or are very volatile, such as zinc and cadmium in copper alloys. The electric arc furnace, which was originally used for all melting operations, did not conform with the increasingly severe requirements in this direction, since a marked volatilisation of the volatile constituents was found to occur at the electrodes whose temperature was about 4000 deg C. It thus became necessary to build a new melting furnace, which had to meet the following requirements:

1) The furnace must take a charge of 50 kg of iron, and also enable scrap and other waste materials to be utilised;
2) The melting charge must be kept free from all impurities; any contact with combustion gases or electrodes was therefore undesirable.
3) To reduce the volatilisation of volatile constituents, temperatures above the casting temperature must be avoided as far as possible. For the same reason the melting time must not exceed 15 minutes.

To meet these requirements, the method of eddy-current heating was adopted, i.e. the heating of the melting charge by electrical eddy currents which are produced in the charge itself by alternating magnetic fields.

In this process the melting charge does not in fact come in contact with either combustion gases or electrodes. In addition, the fusing metal constitutes the hottest part of the furnace so that no temperatures higher than the melting temperature occur.

The first attempts at utilising eddy current heating for practical purposes were made some 30 years ago (Soc. Schneider, Creuzot and O. Zander, 1905). Fundamental theoretical investigations of the induction furnace were carried out by Northrup[1]), Ribeaud[2]), Wever and Fischer[3]), Burch and Davis[4]), Strutt[5]) and others. Details about the general behaviour of an induction furnace will be given below in connection with the description of the Philips melting plant. It has been found that the efficiency of inductive heating in general increases with the frequency. The smaller the pieces making up the charge, the higher are the frequencies required. If pieces of metal a few centimetres in diameter are to be melted with a satisfactory efficiency, frequencies from 5000 to 10000 cycles are found to be necessary.

A voltage of this high frequency cannot be readily and easily produced with ordinary generators. A transmitting valve was therefore employed, viz, Philips TA 20/250 valve which on correct adaptation can furnish a useful output of 250 kW ($V_{\mathrm{eff}} =$ 14000 volts, $I_{\mathrm{eff}} =$ 18 amps). This output is greater than that required for melting the charge in 15 minutes.

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[1]) Northrup, J. Frankl. Inst. **20**, 240, 1926.

[2]) M. G. Ribeaud, La technique moderne, **19**, No. 8-9, 1929.

[3]) F. Wever and W. Fischer, Inst. f. Eisenforsch. **8**, 149, 1926.

[4]) C. K. Burch and N. R. Davis, Arch. f. El. **20**, 211, 1928.

[5]) M. J. O. Strutt, Ann. d. Phys. **82**, 605, 1927; Arch. f. El. **19**, 424, 1928.

Description of the Induction Furnace

The melting throughput of the plant was required to be 200 kg of steel per hour, for which about 250 kW are drawn from the mains supply. The furnace plant can take a load of 300 kW so that it has an ample rating. The capacity of the melting crucible is 12.5 litres (diameter 20 cm and height 40 cm) which when half charged represent 50 kg of steel. If material is added during the melting process, the total charge can be increased to 100 kg. As the time taken for the settling and removal of the slag and for casting is roughly the same as that required for melting alone, two furnaces have been installed which are connected alternately with the same electrical supply. The latter is therefore in constant service.

The furnaces are so small and easy to manipulate that no casting ladles are required. The charges are poured directly into the moulds placed below the furnaces. In this way operations are continuous and regular, while with larger furnace charges the casting metal, after requiring a longer period for melting, must be distributed through one or more intermediate ladles to the various moulds and in fact with great speed, as otherwise the melt is too hot at the start or too cold at the end of casting.

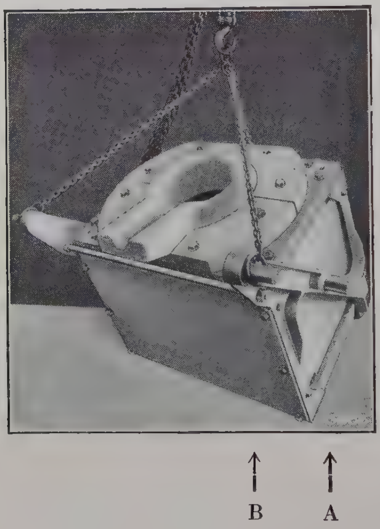

Fig. 1. Induction furnace. On the right-hand wall are the trunnions A and B which support the furnace during service. During heating the furnace rests in A and during casting in B.

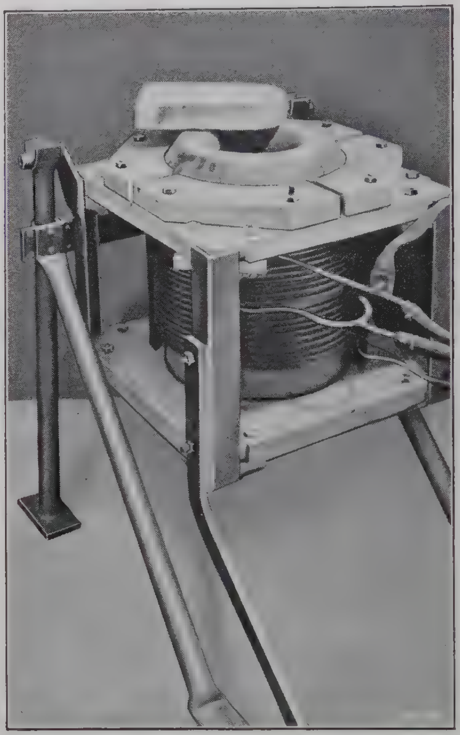

Fig. 2. Induction furnace with side walls removed.

Figs. 1 and *2* show the furnace installed ready for use and with the side walls removed. The base and cover plates of the furnace are made of insulating material and are joined together with brass angles. These plates support the fireproof crucible which is surrounded by an induction coil, being insulated from the latter by an earthenware jacket and a layer of asbestos. The self-induction of the coil is 0.125 millihenry; it is of single-lay and consists of 20 windings of rectangular copper tube, 12.5 by 25 mm, through which cooling water is passed during service. The diameter and height are each 40 cm. In the construction of the furnace, mutual insulation of the metal parts was provided, in order to avoid any short circuits in which powerful eddy currents could be formed. Owing to these precautions the brass units only become hand-warm. Iron which offers a much greater resistance to eddy currents would become red-hot under the same conditions.

Fig. 3 is a general view of the furnace platform and shows the suspension of the furnace during the heating period (left) and during casting (right). During the heating period the furnace is supported by two axle bolts (one of which is visible in fig. 1) which are located just above the centre of gravity, thus enabling the furnace to be easily turned. When pouring the charge, the furnace is turned by the tension of a wire rope. The mouth then moves in a circle forwards and downwards until two other axle bolts, fixed in line with the axis through the mouth, drop into the corresponding bearings.

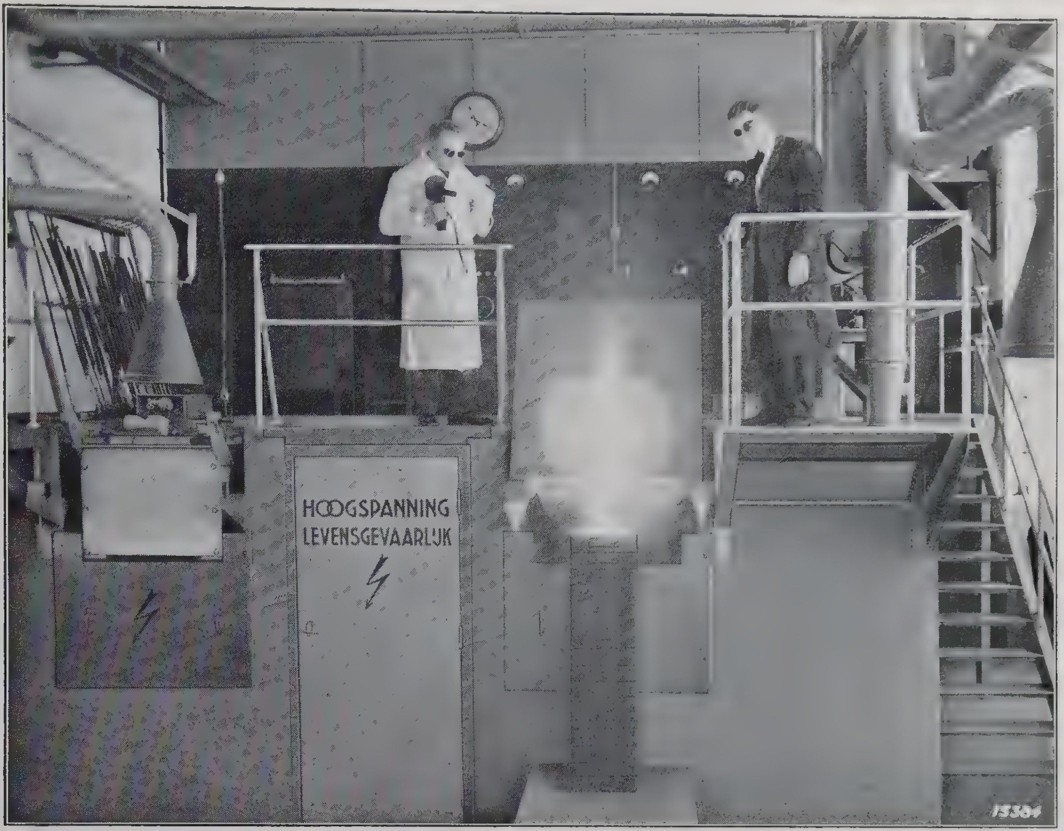

Fig. 3. General view of the melting plant. Behind the furnace is the switchboard for the electrical plant which heats the two furnaces alternatively. The door marked "Hoogspanning" is the entrance to the transformer room.

This position has been reached by the right-hand furnace in fig. 3. With continued tension on the rope, the furnace turns about its mouth so that the stream of poured metal remains in one position and it is unnecessary to move the mould during casting.

The Electrical Plant

Fig. 4 shows the general lay-out of the electrical plant. The latter is made up of a high-frequency generator, an air-core transformer which reduces the voltage from 12500 $\text{volts}_{\text{eff}}$ to 5000 $\text{volts}_{\text{eff}}$, and the condenser C, which takes up the wattless current of the furnace coil. The self-induction of the furnace is represented by L and the non-reactive resistance by R. This non-reactive resistance is made up of the eddy current losses in the charge, the ohmic losses in the coil and the dielectric losses in the condenser. As the oscillating circuit of the valve generator is formed by the furnace coil L and the condenser C themselves, it always operates with the natural frequency of the L-C-circuit. At this frequency the impedance at the poles of the generator is a true non-reactive resistance, thus avoiding an unnecessary loading with reactive currents. The resistance is determined by the type and temperature of the charge, but can also be altered by varying the capacity C[6]).

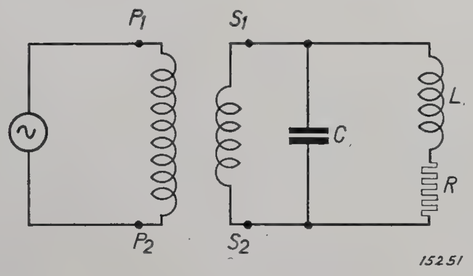

Fig. 4. Electrical lay-out of the melting plant.

This method is employed to maintain the loading resistance of the generator practically constant during the heating of the crucible.

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[6]) This naturally also alters the frequency of the alternating voltage applied to the furnace coil.

Owing to the variability of the loading resistance, the transmitting valve constitutes a marked advance over the mechanical generator. To derive the full output from a current generator the loading resistance must have a definite value. With a mechanical generator, however, the loading resistance cannot be varied in the manner described above because the frequency is generally fixed. It would therefore be necessary to alter the capacity C and the self-induction L simultaneously in such a way that the natural frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

of the oscillating circuit maintains exactly the same value as the frequency of the mechanical generator.

Fig. 3 shows the switchboard of the transmitting-valve plant located at the rear of the platform. The door marked "Hoogspanning" is the entrance to the transformer room which also accomodates a large battery of condensers. *Fig. 5* shows a view of this room as seen from the door, while *fig. 6* shows half of the condenser battery. This battery is made up of 72 separate condensers, each of 0.083 μF, which can be interconnected in four different ways. In the four positions of the switch drum, the following capacities are obtained:

$$0.75\ \mu\mathrm{F},\ 1.5\ \mu\mathrm{F},\ 4.5\ \mu\mathrm{F}\ \text{and}\ 6\ \mu\mathrm{F}.$$

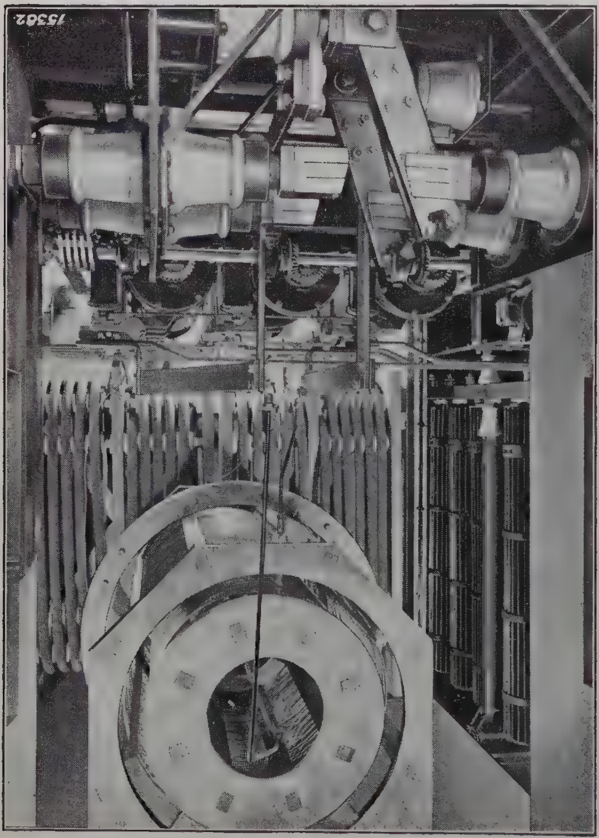

Fig. 5. Transformer of high-frequency furnace. Primary winding (outside) 12500 volts; secondary winding 5000 volts. Part of the battery of condensers may be seen on the right hand wall of the transformer room.

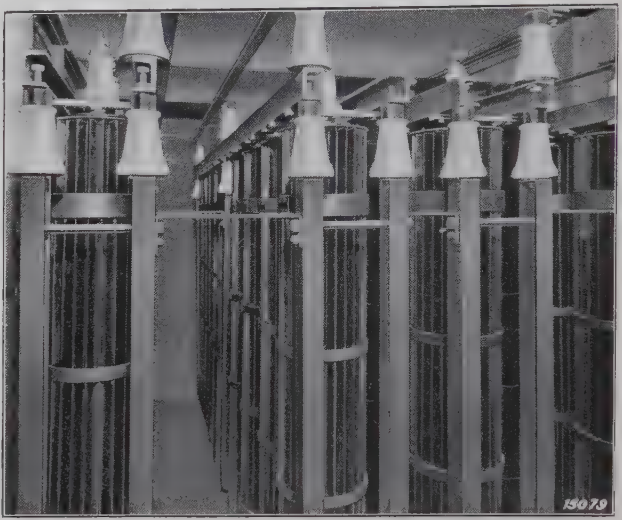

Fig. 6. Half of the battery of condensers. The 72 condensers, each of 0.083 μF, are arranged in 8 series.

The phase difference of the condensers is given by $tg\ \delta = 0.007$. In consequence of the high voltage and frequency of the current the dielectric losses are quite considerable. The dielectric losses N of a condenser are given by:

$$N = \omega C \cdot V_{eff}^{2}\, tg\ \delta = \sqrt{\frac{C}{L}}\, V_{eff}^{2}\, tg\ \delta$$

Substituting the respective numerical values: $V_{eff} =$ 5000 volts, $C = 6$ μF and $L = 0.09$ millihenry, one obtains as a maximum value of the losses

$$N = 45\ \mathrm{kW}.$$

Table I collates the principal data relating to the high-frequency furnace.

Table I. Operating data, relating to the high-frequency furnace.

Capacity of condenser C	Frequency of alternating current
0.75 μF	16300 to 18000 cycles
1.5 μF	11600 to 12800 cycles
4.5 μF	6700 to 7400 cycles
6.0 μF	5800 to 6400 cycles

Self-induction of coil: 0.09 to 0.125 millihenry (depending on the charge).

Voltage applied to furnace coil: . . .	5000 $\mathrm{volts}_{\mathrm{eff}}$
Apparent power	Max. 6400 kVA
Effective power	Max. 200 kW
Useful power in charge:	Max. 130 kW

The required rate of heating is readily attained with the furnace. With a loss of 30 per cent as a result of thermal conduction and radiation, about 18 kW-hrs. are required for melting 50 kg of iron. As under the most favourable conditions of adaptation about 130 kW are furnished to the charge, the time required for melting is found to be 8.3 minutes. The actual time taken is of course longer, but even with the most difficult charge in the crucible only 10 to 15 minutes are required.

In the case of metals with a higher conductivity than iron, the proportion of the total power supplied which is absorbed by the charge is naturally smaller. Nevertheless aluminium, silver and copper can also be melted in this high-frequency furnace. At a first glance this appears surprising, since for instance a charge of copper shavings can in no case be heated to a higher temperature than the copper induction coil itself, if the diameter of the shavings is not greater than that of the copper tube of which the coil is made. This shows quite clearly that water cooling is imperative. Cooling keeps the resistance of the induction coil low, while the resistance of the charge rises as a result of heating to about 4 times its initial value.

Discussion of the Method of Operation of the High-Frequency Furnace

Omitting all consideration of the processes in the high-frequency furnace itself, we can regard it as a loading impedance with an inductance L and a resistance R. For the plant in question we thus obtained the diagram shown in fig. 4. At the resonance frequency, the secondary terminals S_1 and S_2, of the transformer are loaded with a non-reactive resistance W, which in the case of a sufficiently small damping resistance R [7]) of the oscillating circuit, is given approximately by:

$$W = \frac{L}{CR}$$

As already pointed out, the commutable capacity C is so adjusted that the resistance W approaches as close as possible to a specific optimum value W_S. In this case W_S is 127 ohms. The resistance across the primary terminals of the transformer is then $W_P = W_S$ times squared transformation ratio $= 127 \times 6.25 = 720$ ohms.

It is thus equal to the internal resistance of the transmitting valve.

A somewhat closer insight into the process of heat evolution in the furnace is obtained by regarding the furnace coil as the primary winding of a transformer whose secondary winding constitutes the path of the current in the charge. This is shown in the diagram in *fig.* 7 where the indices 1 relate to the furnace coil and the indices 2 to the current path through the charge.

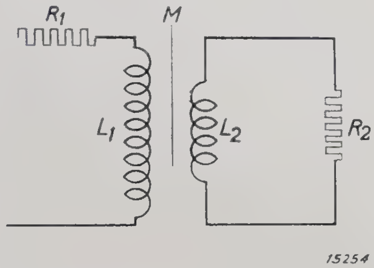

Fig. 7. Substitution diagram for high-frequency furnace. L_1 and R_1 are the self-induction and resistance of the furnace coil. L_2 and R_2 are the self-induction and resistance of the charge. The mutual induction is represented by M.

Owing to the mutual inductance M the resistance of the charge is imparted to the primary coil. In order to obtain a satisfactory efficiency, this resistance R_K which is coupled to the input side must be as large as possible compared with the input resistance R_1. The power input is

$$N = I^2_{\text{eff}} \ (R_1 + R_K)$$

and the efficiency

$$\eta = \frac{R_K}{R_1 + R_K}$$

If ω is the frequency of the alternating current expressed in circular measure, one obtains by a simple calculation

$$R_K = \frac{R_2 \ M^2 \ \omega^2}{\omega^2 \ L_2{}^2 + R_2{}^2} \quad . \quad . \quad . \quad . \quad (1)$$

Equation (1) is derived from the transformer equations:

$$I_1 \ (R_1 + j \ \omega \ L_1) + I_2 \ j \ \omega \ M = U_1$$
$$I_1 \ j \ \omega \ M + I_2 \ (j \ \omega \ L_2 + R_2) = 0,$$

which express the relationship between the primary voltage U_1 and the primary and secondary currents I_1 and I_2. Eliminating I_2 from these equations, one obtains:

$$U_1 = I_1 \left| R_1 + j\omega L_1 + \frac{R_2 \cdot \omega^2 \ M^2}{\omega^2 L_2{}^2 + R_2{}^2} - \frac{j\omega L_2 \cdot \omega^2 \ M^2}{\omega^2 L_2{}^2 + R_2{}^2} \right|$$

The third and fourth terms in this expression give the additional resistance R_K and a corresponding (negative) additional inductance.

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[7]) “Small damping” signifies $\frac{R^2C}{L} \ll 1$.

The variation of R_K with respect to ω is shown in *fig. 8*. This diagram shows $(R_K/R_2) \cdot (L_2^2/M^2)$ plotted against $(\omega L_2/R_2)$.

It is seen from the curve that R_K at first rises quickly with the frequency, eventually to become asymptotic to the limiting value: $R_K \rightarrow R_2\, M^2/L_2^2$.

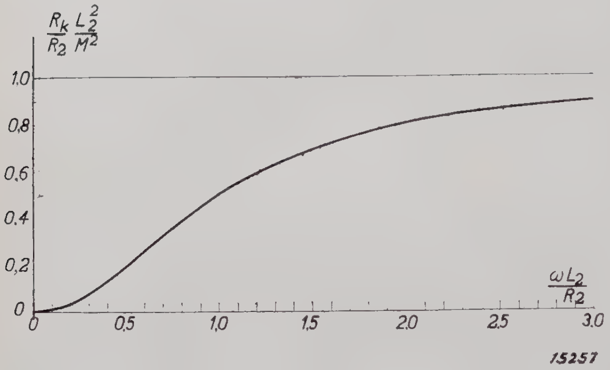


Fig. 8. Coupling resistance R_K plotted against the frequency. When $\omega\, L_2/R_2 = 3$, R_K assumes 90 per cent of its asymptotic limiting value.

As already stated, a high coupling resistance is required in order to obtain a satisfactory efficiency. The frequency must therefore be made so high that the limiting value is almost obtained. On the other hand, it is desirable to keep the frequency as low as possible since at a given output the losses exterior to the furnace as a rule increase with the frequency. We shall therefore stipulate a coupling resistance of only 90 per cent of the limiting value, which according to the graph (8) corresponds to the condition that:

$$\frac{\omega L_2}{R_2} > 3 \quad . \quad . \quad . \quad . \tag{2}$$

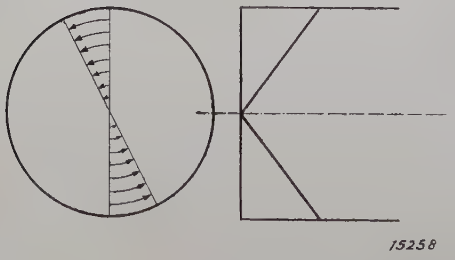


Fig. 9. Intensity of eddy currents produced in a metal cylinder by a homogeneous magnetic field in the direction of the axis. The arrows indicate the direction of the eddy currents.

A further increase in the frequency (in radians) would result in very little improvement of the efficiency.

The substitution of a self-induction L_2 and a resistance R_2 for the electrical properties of the charge asks for a further comment. The distribution of the eddy currents in the charge depends on the frequency of the magnetic field, and as a result L_2 and R_2 also vary with the frequency.

In order to visualise the distribution of the eddy currents, consider firstly the limiting cases at very low and very high frequencies. Consider a cylinder whose axis is parallel to the magnetic field (*fig. 9*). It is well known that the induction currents are in such directions that within the metal they weaken the magnetic field. If the frequency is sufficiently low, the induced voltages and currents are, however, so small that the weakening of the field can be neglected. In this case the distribution of current shown in fig. 9 is obtained. The current density is zero along the axis and increases in proportion to the radius.

The higher the frequency, the greater will be the weakening of the magnetic field in the interior of the cylinder. At the same time the eddy currents in the interior also become weaker and with rising frequency we get in succession the current distribution curves II, III and IV shown in *fig. 10*. In the limiting case of a very high frequency the current flows merely through a thin layer at the surface.

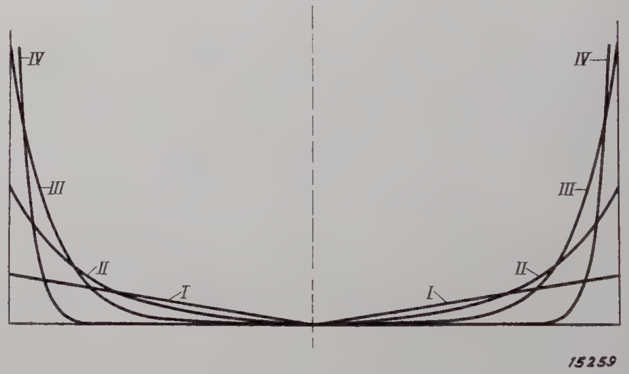


Fig. 10. Intensity of the eddy currents produced in a cylinder at different frequencies. Owing to the natural magnetic field of the eddy currents, the total field is not homogeneous, but greater at the walls than in the interior. With rising frequency the current distribution is altered from I (see also fig. 9) to II, III and IV.

We shall reserve the opportunity of returning to these interesting phenomena in greater detail in a subsequent issue of this periodical. Closer investigation shows that the condition $\omega L_2/R_2 > 3$ in every case leads to such a high frequency that the displacement of the magnetic field from the interior of the pieces making up the charge already becomes very pronounced. The current flows through a layer at the surface, whose thickness at a given frequency is only slightly dependent on the sizes of the pieces. In this case the fraction of the self-induction value L_2 due to a single piece (e.g. of spherical shape) will increase with the volume, while the resistance R_2 will be practically independent of the size of the piece. As a result the ratio L_2/R_2 will increase with the size modulus. The smaller the pieces making up the charge, the higher therefore will be the frequency required in order to make $\omega L_2/R_2$ sufficiently large.

With the frequencies of 6000 to 7000 cycles used here, the condition $\omega L_2/R_2 > 3$ is still met with pieces a few centimetres in size. In general the policy will be not to use the frequencies higher than are absolutely necessary, as the generation of high-frequency alternating currents with a satisfactory efficiency becomes progressively more difficult as the frequency is raised (e.g. owing to the increase in the dielectric losses).

Convection Currents in the Melt

A particularly valuable characteristic of the induction furnace is the production of convection currents in the melt by the action of electromagnetic forces, which promote a rapid and uniform intermixing of the alloying constituents. The magnetic field acts on the conductor of eddy currents with a gravitational force directed inwards (the eddy currents are in a direction opposite to the current through the coil). The pressure in the fluid mass therefore increases towards the centre. *Fig. 11* shows diagrammatically the distribution of pressure in the melting crucible, which curve can also be regarded as a contour of the free surface. As near the surface of the melt the magnetic field is not homogeneous, the pressure caused by the magnetic forces will vary with the height. This leads to the formation of convection currents, which are marked qualitatively in fig. 11. With eddy currents of a given intensity, the pressure causing the convection currents is proportional to the magnetic field. In a practical sense this signifies that convection decreases with rising frequency, for the greater the frequency, the weaker are the magnetic fields required to generate the requisite eddy currents.

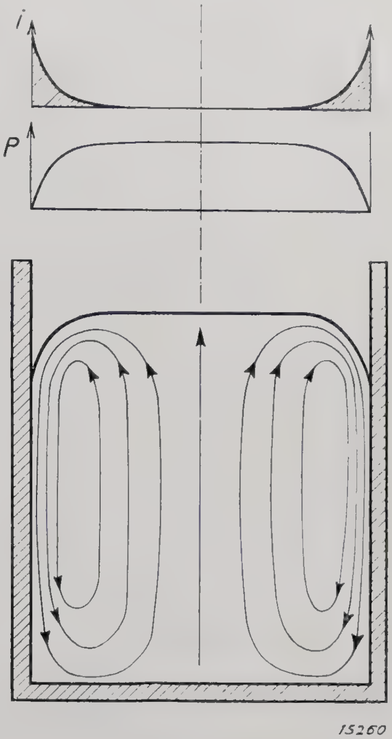


Fig. 11. Convection currents in the molten metal. Above: first curve, current distribution i; second curve: pressure P, produced by magnetic forces.

Compiled by G. HELLER.

PRACTICAL APPLICATIONS OF X-RAYS FOR THE EXAMINATION OF MATERIALS

II.

By W. G. BURGERS.

Two examples are given here of the examination of materials by means of X-rays in which the difference in the properties of the preparations examined is not due to a difference in chemical composition, but to a difference in texture, i.e. in the arrangement of the crystals of which the substances are built up.

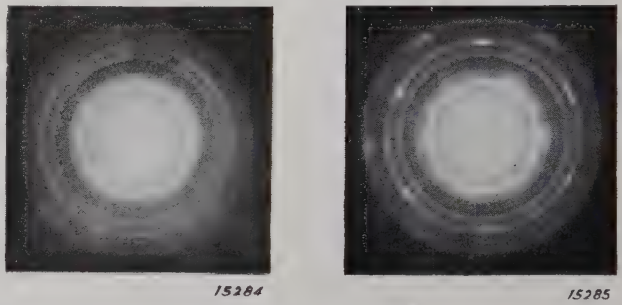
The "scattering" of X-rays by crystals may be visualised as a reflection of the rays at the planes which are formed by the atoms in the crystal; from this it may be deduced that a substance in which the crystallites do not assume a preferred orientation gives rise to lines in the X-ray pattern which are uniformly black throughout their whole length. If, on the other hand, the crystallites have a certain preferred orientation (as is always found for instance in worked metals and also in many natural substances), the rays are reflected more strongly in certain directions than in others, and hence the blackening of the X-ray lines varies along their length.

In the limiting case where the rays impinge on only one crystal, this effect may be so pronounced that the lines are resolved into discrete points, whose arrangement then corresponds to that of the planes in the crystal lattice, i.e. to the symmetry of the crystal (Laue-photograph).

3. Quality Tests on Soapstone before Firing [1]).

Soapstone is a soft material which can be worked to exact dimensions. After suitable shaping it can be converted by firing (heating to about 1200 deg. C, during which about 6 per cent of water is given off) into a compact, hard and heat-resisting stone. During firing it frequently happens that certain places swell and as a result cracks appear which make the product useless. It is desirable to be able to detect and reject these pieces before firing, and as it appeared probable that the cracking was associated in some way with certain structural characteristics, examination by means of X-rays was indicated as offering a possible solution.

Fig. 1a shows a radiograph of that part of the raw material which did not swell on subsequent firing, while *fig. 1b* is a similar exposure of another part where swelling actually took place later.



a) b)

Fig. 1. Radiographs of steatite before firing:
a) of a place which did not swell on subsequent firing, and
b) of a place which later swelled on firing.

An examination of these exposures reveals the same system of interference rings [2]) in both. But there is a fundamental difference in that the intensity distribution is uniform round the periphery of the rings in exposure 1a, but varying in 1b, where certain intensity maxima appear round the circumference. This enables us to conclude that while there is no essential difference in composition at swelling and non-swelling places (identical systems of rings are obtained) there is yet a difference in texture: the places swelling on subsequent firing reveal that the crystallites favour a pronounced directional configuration (fibro-crystalline structure). Thus without entering into an analysis why this difference in structure causes a swelling during firing, we have evolved a method for testing the quality of soapstone before the firing process.

4. Differentiation between Natural and Cultured Pearls

A natural pearl is built up of concentric layers of calcium carbonate (mother-of-pearl) which have been deposited by the oyster round any available nucleus (e.g. a grain of sand). Each layer consists of crystallites with a sixfold symmetry, the axis of each crystallite being perpendicular to the layer. This structure is shown schematically in *fig. 2a*.

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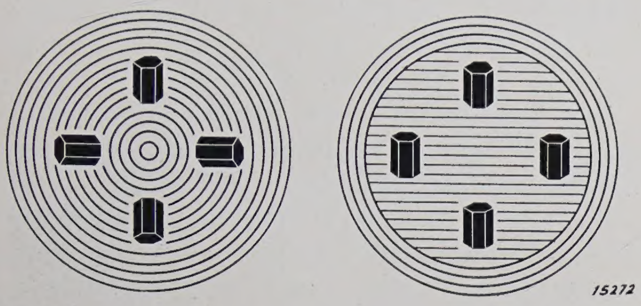
[1]) W. G. Burgers and J. Hoekstra, Polytech. Weekblad **29**, 443, 1935.

[2]) That here complete circles appear, while in the reproductions included in the first article (cf. this Review **1**, 29, 1936) the lines consisted only of circular arcs, is due to the method of registration. If it is only required to determine the relative positions of the rings, it is sufficient to use a narrow strip of film cut along a diameter of the circles.

A "cultured" (or Japanese) pearl is obtained by inserting in the oyster a bead of mother-of-pearl cut from an oyster shell. This shell is built up of plane layers of mother-of-pearl with the sixfold axis of symmetry of the individual crystallites again arranged perpendicular to the layer. The nucleus of a cultured pearl therefore has a structure as shown schematically by the horizontally shaded section in *fig. 2b*. Round this "nucleus" the oyster deposits several thin concentric layers so that in external appearance the finished cultured pearl cannot be distinguished from a natural pearl.

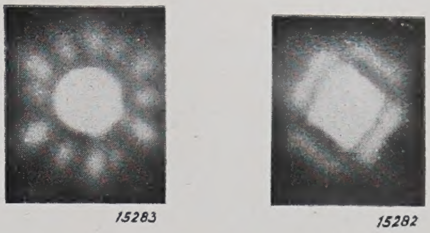


a) b)

Fig. 2. Diagrammatic representation of the structure of a natural and a cultured pearl.
a) The natural pearl is made up of concentric layers of mother-of-pearl (crystallites of calcium carbonate).
b) The cultured pearl consists of a nucleus of plane layers of mother-of-pearl, on the outside of which several thin concentric layers have become secreted.

If now a narrow beam of X-rays is brought to impinge on a natural pearl in any direction, the rays will always be perpendicular to the layers, i.e. parallel to the axis of the hexagonal crystals of calcium carbonate. On the other hand with a cultured pearl, the rays will as a rule not coincide with the direction of the crystal axis [3]) (the external concentric layers are so thin that compared to the pearl as a whole their effect can be neglected).

From the connection between the symmetry of the crystals and the X-ray pattern (which is produced by reflection of the rays at the crystal lattice planes) referred to at the beginning, it follows that the patterns obtained with a natural pearl must always exhibit an arrangement of dots with a sixfold symmetry, which latter cannot be obtained with a cultured pearl. *Figs. 3a and 3b* confirm this conclusion, and show that it is indeed possible to distinguish between natural and cultured pearls by means of X-rays [4]).



a) b)

Fig. 3. a) The diffraction pattern of a natural pearl always exhibits a sixfold symmetry.
b) A cultured pearl gives another type of pattern.

This method, together with other (e.g. optical) methods, is employed in practice by jewellers. For this purpose a special apparatus has been developed, which is shown in *fig. 4*.

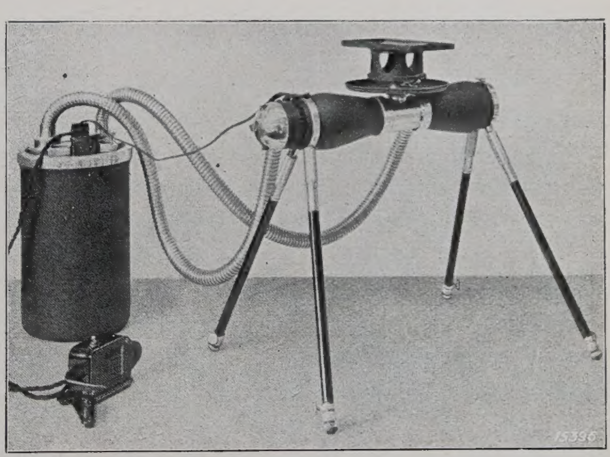


Fig. 4. The Philips pearl-testing apparatus. On the right hand is the "Metalix" X-ray tube mounted horizontally on four legs. On the tube the camera is fixed containing the support for the pearls to be examined. At the rear on the left is the high-tension transformer, in front of it the switch.

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[3]) Such coincidence would only be obtained if the X-rays accidentally impinged on the pearl in a vertical direction in the figure. A second exposure made after slightly turning the pearl would, however, soon bring this to light.

[4]) The principle of this method has been indicated by J. Galibourg and F. Ryziger, Revue d'Optique **6**, 97, 1927.

SHORT NOTICES

Dosage control in X-ray Laboratories and Works

X-rays can exercise a healing action on diseased tissues as well as cause serious injury to healthy tissues. In both cases a specific "dosage" of rays is necessary before the respective effects are produced. Originally the dosage of X-rays was indeed defined on the basis of their biological action, an exposure to the radiation being taken as a unit dose, viz, 1 HED, when it just sufficed to produce an erythema (sunburn) on the human skin. Today this biological definition of dosage has been replaced by a physical one which is more accurately reproducible and which is based on the ionisation action of the X-rays (air being rendered an electrical conductor). This physical unit is the "Röntgen" (abbreviated "r"), and is equivalent to 1/600 HED.

Where a person working with X-rays is exposed to accidental (stray) radiation, it has been laid down that the dosage of exposure must not exceed 0.2 r per day. Experience has shown that when a person is exposed to this maximum radiation even over a long period no ill effects need be feared. In modern X-ray tubes, such as the Philips "Metalix" tube, care has been taken that outside the useful beam of rays the dosage is always below this limiting or tolerable dose. There is thus no danger whatsoever outside the actual X-ray beam in the use of all permanent X-ray installations designed for specific purposes, such as diagnosis, therapy or the radiographic examination of materials. But in laboratories and in works making X-ray apparatus fairly frequent exposure of the research worker or the workman to the action of the rays cannot be avoided. Naturally in these cases one must not wait until the exposed person begins to show the symptoms of harmful exposure to the rays before providing some form of special protection: it is essential to test continually whether those working in such exposed positions are being exposed to a daily dosage below the tolerable maximum of 0.2 r or not.

In the Philips X-ray laboratory and works, this is done by providing each person with a small cassette containing an X-ray film, which is always carried in the pocket. Every week this film is developed and the intensity of blackening measured photometrically, from which is calculated the X-ray dosage which has fallen on the film and to which therefore the person carrying the film has been exposed. The blackening of the film is not fully determined by the dosage, but is also dependent on the hardness (wave-length) of the X-rays; this can be taken into consideration in a simple way.

In many occupations it is natural that not the whole body of the exposed person receives the same dosage. Thus, in the case of a physician who frequently must place his hands in the useful beam of X-rays, these will be exposed to the action of the rays far more than any other part of the body. In this case a small film is carried on the hand or wherever the greatest exposure to the rays may be expected.

Brightness greater than on the sun

Brightnesses greater than those on the sun were recently produced in the Philips Laboratory by further increasing the wattage of the water-cooled super-high-pressure mercury vapour lamp described by Bol (De Ingenieur **50,** E 91, 1935). Theoretical considerations of Elenbaas indicated that by reducing the diameter the brightness would be increased, and a lamp was therefore constructed with an internal diameter of 1 mm and an external diameter of $3^1/_2$ mm, which with electrodes 10 mm apart and an 805-volt alternating current took a load of 1400 watts. The luminous intensity with this load was 11,000 candle-power and the pressure about 200 atmos. Along the axis of the discharge the brightness was 1,160,000 candle-power per sq. in.; this was measured by passing a photo-electric cell with a small slit parallel to the axis of the discharge tube across an enlarged image of the lamp.

Seen from the earth, the brightness at the surface of the sun is only 1,065,000 candle-power per sq. in. (according to the International Critical Tables).

ABSTRACTS OF RECENT SCIENTIFIC PUBLICATIONS OF THE N.V. PHILIPS' GLOEILAMPENFABRIEKEN

No. 1036: Balth. van der Pol, Interaction of radio waves, II (T. Ned. Radio Genootsch. **7,** 93-97, Sept. 1935).

This report contains the conclusions drawn from 1823 observations of the interference produced in the modulation of radio waves reflected by the Heaviside layer. These observations which were made by 30 radio workers in different parts of the world relate primarily to the interference with reception of a number of European radio stations caused by a Morse signal specially radiated for this purpose by the Luxemburg transmitter. It is found that the effects produced by a station with a long wave-length on transmitters of long and medium wave-lengths are of the same order of magnitude. The effect has a monotonic decrement as the distance increases between the interfering station and a point midway between the station searched for and the observer. If this distance exceeds 600 km, interference is practically negligible. The observations made show that interference is due to the non-linear conductivity of the ionosphere, and also give a closer insight into the manner radio waves are reflected at the ionosphere.

No. 1037: J. A. M. van Liempt, Die Berechnung der Auflockerungswärme der Metalle aus Rekristallisationsdaten (Z. Phys. **96,** 534-541, Sept. 1935).

A formula is derived for calculating the energy required for disintegrating the crystal lattice of metals from recrystallisation data. This energy expressed in cals. per gram-atom is approximately 4.6 times the socalled recrystallisation constant; more roughly still, this energy is proportional to the melting point, the factor of proportionality being 32 cals. per degree. This relationship is in close agreement with observation. In conclusion the author discusses Langmuir's and his own diffusion formulae, referring inter alia to various still obscure diffusion phenomena.

No. 1038*): J. G. C. Stegwee: Het harden van snelstaal met hoog kobaltgehalte (Metaalbewerking **2,** 341-343, Sept. 1935).

A practical method for hardening a high-speed tool steel containing 18 per cent tungsten and 15 to 18 per cent cobalt is described. It was found, inter alia, that in tools which have been hardened in oil from temperatures above 1300 deg. C, fracture immediately took place as a result of hardening cracks. This was avoided by hardening in compressed air. In order to obtain adequate hardening, it is essential to anneal twice at 620 to 630 deg. C for 15 to 30 minutes each time. A single annealing for a longer period is not sufficient.

No. 1039: E. J. W. Verwey: The structure of the electrolytical oxide layer on aluminium (Z. Kristallogr. **A 91,** 317-320, Sept. 1935).

A structure is proposed for cubic γ'-Al_2O_3 obtained by electrolytic oxidation of aluminium. The elementary cell according to this structure contains 4 negative O ions in a space-centred configuration. Of the positive Al^{3+} ions $2^2/_3$ are present in each elementary cell, i.e. they are distributed statistically over all the spaces between the O^{2-} ions, and in such a way that 70 per cent of the Al^{3+} ions have the co-ordination number 6 with respect to oxygen and 30 per cent the co-ordination number 4. The modifications γ'-Al_2O_3 and γ-Al_2O_3 represent intermediate cases between the amorphous state and that with a complete atomic arrangement.

No. 1040: E. J. W. Verwey: Incomplete atomic arrangement in crystals (J. chem. Phys. **3,** 592-593, Sept. 1935).

With respect to their positive ions, γ-Fe_2O_3, γ-Al_2O_3 and γ'-Al_2O_3, which is produced on electrolytic oxidation of Al, possess a structure which can only be defined statistically. The two modifications of γ and γ' have indeed the same oxygen lattice, although they differ in the incompleteness of the arrangement of the negative ions. These structures represent transition stages between the amorphous state and the complete atomic arrangement.

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*) A sufficient number of reprints for purposes of distribution is not available of those publications marked with an asterisk (*). Reprints of other publications may be obtained on application from Philips Laboratory, Eindhoven, Holland.

No. 1041: J. A. M. van Liempt: De wet voor overeenstemmende toestanden van roosterherstel (Chem. Wbl. **32**, 546-550, Sept. 1935).

If a piece of metal which has been worked or shaped is heated to a sufficiently high temperature for some time, a "recovery" of its lattice takes place. On heating for different periods (*t* secs.) to different temperatures (*T* deg. abs.) "corresponding states" of lattice recovery are obtained, if $T\,(13.5 + \log t)$ is a constant. This formula is in satisfactory agreement, for instance in the case of brass and pure copper, with practical experience.

No. 1042: P. G. Cath: Het gedrag van metaaldraadlampen bij het branden op wisselstroom (Ingenieur **50**, E 101-104, Aug. 1935).

The temperature of the metal filament fluctuates when electric lamps are run from an alternating current supply. These fluctuations are calculated in this paper and represented in a diagram. In addition it is stated for a certain mean temperature how many times the vaporisation, the luminous intensity and thermal emission of electrons from the filament is greater with alternating current than the corresponding values for direct current.

No. 1043: K. F. Niessen: Erweiterung einer früheren Formel für die Erdabsorption in der drahtlosen Telegraphie (Ann. Physik **24**, 31-48, Sept. 1935).

The author calculates the absorption by the earth of radiated wireless waves for various types of surface strata (sea water, fresh water, wet and dry soil) and for various dipole levels above the surface of the ground, the latter within certain limits. It is shown that the wave-lengths may also assume values in which the densities of the displacement and conduction currents in the earth are in any arbitrary ratio.

No. 1044: J. Sack: A new method for the investigation of the transfer of material through the welding arc (Symposium on the welding of iron and steel, organised by the Iron and Steel Institute, London, May 1935, vol. **2**, 553-559).

The drop transfer from the welding electrode to the work-piece is registered by means of the slow-motion camera. For this purpose the welding process is illuminated with X-rays. Compared to exposures with ordinary light, these rays offer the advantage that the formation of metal drops within the coating of the electrode can be recorded. In addition, the definition of the pictures is not adversely affected by the clouds of gas and vapour surrounding the welding arc. (See the short article on this subject in No. 1 of this Review, p. 26).

No. 1045: J. A. M. van Liempt and J. A. de Vriend: Eine einfache Methode zur Bestimmung der Farbtemperatur von Blitzlichtern (Z. wiss. Photogr. **34**, 237-240, Oct. 1935).

The colour-scale temperature of flashlights cannot be determined by the ordinary spectro-photometric method, as their duration is too short. It is therefore necessary to employ exposures of Lagorio's colour scale on panchromatic plates, obtained with both flashlights and different light-sources of known colour-scale temperature. A colour-scale temperature of approx. 4000 deg. abs. is found for magnesium, aluminium and alloys of these two metals burning in oxygen.